Commutative, associative and distributive properties

When you swap two terms, move parentheses, or multiply out parentheses, how do you know the expression is still worth the same, for every number the letters could be?

WORDS TO KNOW

Real number: any number on the number line. $\sqrt{2}$, -3 and 0.4 are all real numbers.

ab: a times b. Two letters side by side are multiplied.

Commutative: about order. You may swap two numbers.

Associative: about grouping. You may move the parentheses.

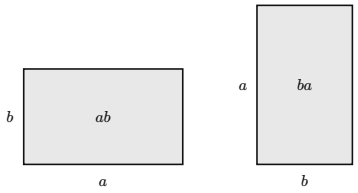
Distributive: a factor is handed out to each term of a sum.

Counterexample: one choice of numbers that shows a rule is false.

KEY IDEAS

1. Order does not matter for adding or multiplying

Turn a rectangle on its side and its area stays the same. Lengths laid end to end reach the same point in either order.



$$a + b = b + a \qquad ab = ba$$

2. Grouping does not matter for adding or multiplying

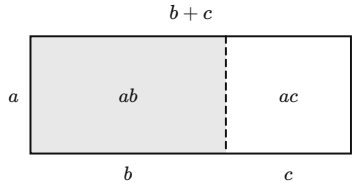
The numbers keep their order. Only the parentheses move.

$$(a + b) + c = a + (b + c) \qquad (ab)c = a(bc)$$

Tip: did the order change? Commutative. Did only the parentheses move? Associative.

3. Multiplication spreads over a sum

The whole rectangle has area $a(b + c)$. Its two parts have areas ab and ac.



$$a(b + c) = ab + ac \qquad a(b - c) = ab - ac$$

Left to right it **expands**. Right to left it **factors**: $ab + ac = a(b + c)$.
Combining like terms is the same rule: $5x + 3x = (5 + 3)x = 8x$.

WATCH OUT

- **Subtraction and division have neither property.** $10 - 4 = 6$ but $4 - 10 = -6$, and $12 \div 3 = 4$ but $3 \div 12 = 0.25$. One counterexample is enough.
- **A term moves with its sign.** Think of $a - b$ as $a + (-b)$. So $5t - 8 + 2t = 5t + 2t - 8 = 7t - 8$.
- **Hand the factor to every term.** $3(x + 4) = 3x + 12$, not $3x + 4$. Never hand it to both factors of a product: $2(5 \cdot 7)$ is 70, not $10 \cdot 14$.

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WORKED EXAMPLES

Example 1. Name the property that makes each statement true.

1. $(4 + n) + 1.5 = 4 + (n + 1.5)$. The order 4, n, 1.5 stays and only the parentheses move: the associative property of addition.
2. $k\sqrt{3} = \sqrt{3}\,k$. The two factors swap places: the commutative property of multiplication.
3. $5(2r - 3) = 5(2r) - 5 \cdot 3$. The 5 is handed to each term: the distributive property.

Example 2. Work out (3.7 + 48) + 6.3 without a calculator.

1. 3.7 and 6.3 make a whole number, so bring them together. Swap inside the parentheses (commutative property of addition): (48 + 3.7) + 6.3.
2. Move the parentheses (associative property of addition): 48 + (3.7 + 6.3).
3. Add: 48 + 10 = 58.

Answer: 58.

Example 3. Simplify $7y + 4(y - 3)$, giving a reason for each step.

Expression	Reason
$7y + 4(y - 3)$	
$= 7y + (4y - 12)$	distributive property
$= (7y + 4y) - 12$	associative property of addition (-12 keeps its sign)
$= (7 + 4)y - 12$	distributive property, right to left
$= 11y - 12$	arithmetic

Answer: $11y - 12$. **Check:** at $y = 1$, 7 + 4(1 − 3) = 7 − 8 = −1, and 11 − 12 = −1.

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