Sets: elements and membership

When you gather things into one collection, what does it mean for a thing to belong to it, and when are two collections the same?

WORDS TO KNOW

Set: a collection of objects, often listed between braces, like $A = \{2, 4, 6, 8\}$.

Element (or member): one of the objects in a set.

The sign $\in$: read "is an element of". So $4 \in A$ says 4 is in A.

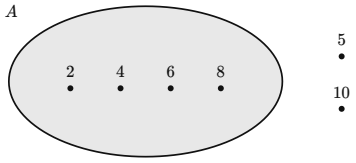
The sign $\notin$: read "is not an element of". So $5 \notin A$ says 5 is not in A.

Empty set: the set with no elements, written $\{\ \}$ or $\varnothing$.

Equal sets: two sets with exactly the same elements.

KEY IDEAS

1. Belonging is a yes or no question



Picture a set as a loop. Each element is a dot inside it, and anything outside is not an element. For any object, the answer to "is it in the set?" is yes or no, never "sort of".

For any object x and any set A, exactly one of these is true:

$$x \in A \qquad x \notin A$$

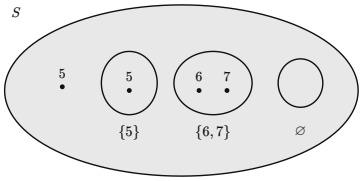
2. A set is decided only by its elements

Two sets are equal when every element of each one is also an element of the other. The order you list them in, and how many times you write one, are not part of the set.

Order does not matter: $\{2, 5\} = \{5, 2\}$.

Repeats collapse: $\{7, 7\} = \{7\}$. The number of elements counts *different* objects.

3. A set can be an element of another set



The elements of a set are the things written directly between its outer braces, split by commas. A set inside a set counts as one element, and what is inside that inner set is one layer down.

Membership counts one layer only. In $\{1, \{2, 3\}\}$, the set $\{2, 3\}$ is an element, but 2 is not.

WATCH OUT

- **A set needs a clear yes or no for every object.** "The good movies" is not a set, because "good" has no cutoff. "Movies longer than 2 hours" is a set.
- **A number is not the set that holds it.** The number 4 and the set $\{4\}$ are different objects.
- **The empty set is a real set.** $\varnothing$ has no elements, but it is still one object. So $\{8, \varnothing\}$ has two elements: 8 and $\varnothing$.

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WORKED EXAMPLES

Example 1. Let $M = \{3, 8, 3, 11, 8\}$ and $N = \{11, 3, 8\}$. How many elements does each have? Is $M = N$?

1. List the different objects in M: 3, 8 and 11. The second 3 and second 8 add nothing.
2. List the different objects in N: 11, 3 and 8.
3. Both hold exactly 3, 8 and 11, and nothing else. Order does not matter.

Answer: each has 3 elements, and $M = N$.

Example 2. Let $K = \{7, \{7\}, \{2, 9\}\}$. True or false: $7 \in K$, $9 \in K$, $\{9, 2\} \in K$.

1. Read the elements between the outer braces: the number 7, the set $\{7\}$ and the set $\{2, 9\}$. So K has 3 elements.
2. $7 \in K$ is true: 7 is on that list.
3. $9 \in K$ is false: 9 is inside $\{2, 9\}$, one layer down, not on the list.
4. $\{9, 2\} \in K$ is true: $\{9, 2\} = \{2, 9\}$, since order does not matter.

Answer: true, false, true.

Example 3. Find every value of x for which $\{x, 4\} = \{4, 3x - 10\}$.

1. The element x on the left must be on the right, so $x = 4$ or $x = 3x - 10$.
2. Try $x = 4$. The left set is $\{4, 4\} = \{4\}$. The right set is $\{4, 3(4) - 10\} = \{4, 2\}$. The 2 is not on the left, so this fails.
3. Solve $x = 3x - 10$: subtract $3x$ to get $-2x = -10$, so $x = 5$.
4. Try $x = 5$. The left set is $\{5, 4\}$. The right set is $\{4, 3(5) - 10\} = \{4, 5\}$. Same elements.

Answer: $x = 5$ only. **Check:** 3(5) – 10 = 5, so both sets are $\{4, 5\}$.

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