Fermi estimation: big numbers from small guesses

Nobody has counted how far you walk in a year. Break a question like that into small guesses you can make, multiply them with their units, and you get an answer worth trusting.

WORDS TO KNOW

Fermi estimate: a rough value built by multiplying smaller quantities you can guess.

Order of magnitude: the power of ten when a number is written as $a \times 10^n$ with $1 \le a < 10$. For $700 = 7 \times 10^2$ it is 10^2.

Bounds: a low and a high estimate that fence in the answer.

Assumption: a guessed number you state out loud, so anyone can check it or change it.

Agree to an order of magnitude: the larger estimate is less than 10 times the smaller.

$\approx$: "is approximately equal to."

KEY IDEAS

1. Build a chain of guesses, with units

Write the big number as a product of small ones you can guess: how many, how often, how long. State every assumption. Write each factor with its units, so you can see them cancel.

$$Q \approx q_1 \times q_2 \times \cdots \times q_k$$

The units must cancel to leave the unit of Q. If they do not, a factor is missing or upside down.

2. Keep one or two digits and judge by the power of ten

Your guesses are good to one or two significant figures at most, so the answer is too. What matters is the power of ten: millions, not thousands and not billions.

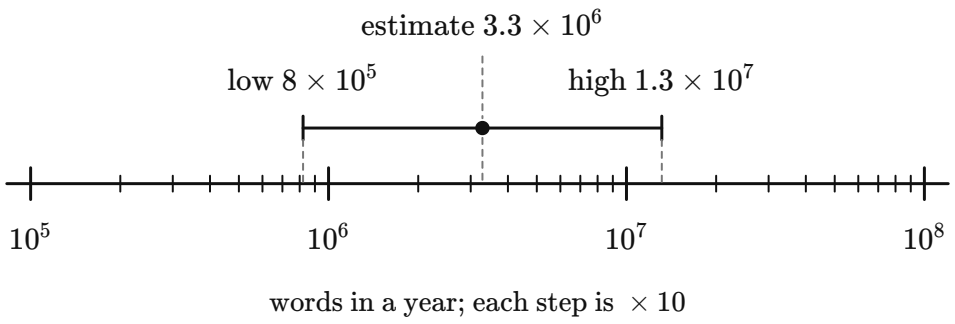
Round to one or two significant figures and write $a \times 10^n$.

3. Fence it in with bounds, then take the middle

On a power-of-ten scale each step multiplies by ten. The middle of a range is the value that is the same number of times above the low end as it is below the high end.

Low bound: multiply the low guesses. High bound: multiply the high guesses. A factor that divides swaps: its high guess gives the low bound.

The middle of a range from L to H is $M = \sqrt{LH}$, because $\dfrac{M}{L} = \dfrac{H}{M}$.



Free sample from math.wiki for classroom use. Do not post. math.wiki

WATCH OUT

- **Do not keep every digit.** A calculator answer claims more than your guesses know. Round it.
- **The ordinary average is not the middle.** Between 10 and 1000 the average, 505, sits near the top. The middle is $\sqrt{10 \times 1000} = 100$.
- **Errors cancel only when you guess each factor on its own.** If every guess leans the same way, the errors pile up.

WORKED EXAMPLES

Example 1. Estimate how many kilometers you walk in a year.

1. Assume you take about 6000 steps a day, and each step is about 0.75 m.
2. Steps cancel, leaving meters a day:
$$\frac{6000 \text{ steps}}{1 \text{ day}} \times \frac{0.75 \text{ m}}{1 \text{ step}} = 4500 \ \frac{\text{m}}{\text{day}}$$
3. Days cancel, then meters cancel, leaving kilometers a year:
$$4500 \ \frac{\text{m}}{\text{day}} \times \frac{365 \text{ days}}{1 \text{ year}} \times \frac{1 \text{ km}}{1000 \text{ m}} = 1642.5 \ \frac{\text{km}}{\text{year}}$$
4. Round to two significant figures.

Answer: about 1.6×10^3 km a year, order of magnitude 10^3.

Example 2. About how many people can stand in a gym 28 m long and 15 m wide? Give bounds and a middle.

1. The floor is 28 × 15 = 420 square meters. Assume each person needs between 0.2 and 0.8 square meters.
2. People = floor area ÷ space per person. The space divides, so the low bound uses its high guess, and the high bound its low guess:
$$\text{low: } \frac{420}{0.8} = 525 \qquad \text{high: } \frac{420}{0.2} = 2100$$
3. The middle: $\sqrt{525 \times 2100} = \sqrt{1\,102\,500} = 1050$.

Answer: about 1.1×10^3, roughly a thousand people, between about 530 and 2100. **Check:** 1050 is 2 times 525 and half of 2100.

Free sample from math.wiki for classroom use. Do not post. math.wiki