Identity and inverse elements

Is there a number that changes nothing when you add it or multiply by it, and can the effect of every number be undone?

WORDS TO KNOW

Identity: a number that changes nothing. 0 for adding, 1 for multiplying.

Opposite (additive inverse): $-a$, the number that adds to a to make 0.

Inverse: a number that undoes another one and brings you back to the identity.

Reciprocal (multiplicative inverse): $\frac{1}{a}$, the number that multiplies by a to make 1.

KEY IDEAS

1. Adding 0 or multiplying by 1 changes nothing

For every number a, adding 0 leaves a as it was, and so does multiplying by 1.

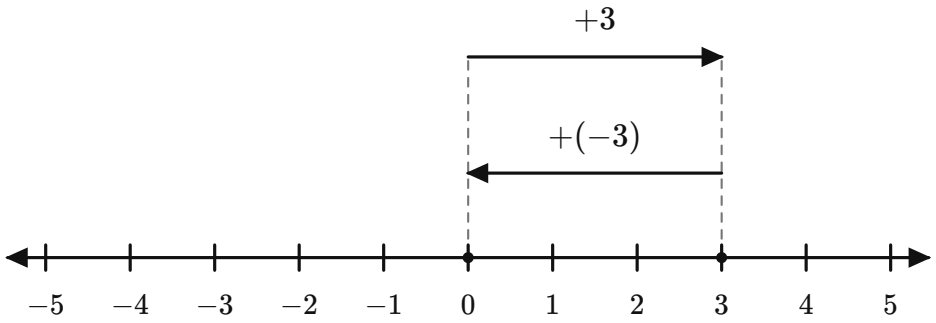
$$a + 0 = 0 + a = a \qquad a \cdot 1 = 1 \cdot a = a$$

2. The opposite undoes adding

Adding 3 is a jump of three steps right. Adding −3 jumps back. The two jumps together do what adding 0 does.

$$a + (-a) = 0$$

Every number has exactly one opposite. The opposite of 0 is 0.



3. The reciprocal undoes multiplying

Multiplying by 3 sends 2 to 6. Multiplying by $\frac{1}{3}$ sends it back. To find a reciprocal, write the number as a fraction and turn it over. Keep the sign.

$$a \cdot \frac{1}{a} = 1 \quad (a \neq 0) \qquad \text{the reciprocal of } \frac{p}{q} \text{ is } \frac{q}{p}$$

0 has no reciprocal: 0 times any number is 0, never 1.

4. Subtracting and dividing go through inverses

To solve an equation, undo what was done to the letter, last step first. Adding is undone by the opposite, and multiplying by the reciprocal.

$$a - b = a + (-b) \qquad a \div b = a \cdot \frac{1}{b} \quad (b \neq 0)$$

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WATCH OUT

- **A negative number has a negative reciprocal.** The product must be positive 1, so the reciprocal of −6 is $-\frac{1}{6}$, not $\frac{1}{6}$.
- **Turn the whole number over.** Write a mixed number or a decimal as one fraction first. Flipping only the fraction part does not give 1.
- **Never multiply both sides by 0.** You get 0 = 0, true for every number, and the step cannot be undone. Any other number is safe.

WORKED EXAMPLES

Example 1. Find the opposite and the reciprocal of $-\frac{6}{11}$, 0.65, $3\frac{1}{4}$ and 0.

1. Opposites: change the sign. They are $\frac{6}{11}$, −0.65, $-3\frac{1}{4}$ and 0.
2. Reciprocals: write each number as one fraction, turn it over, and keep the sign.

$$-\frac{6}{11} \text{ gives } -\frac{11}{6} \qquad 0.65 = \frac{65}{100} = \frac{13}{20} \text{ gives } \frac{20}{13} \qquad 3\frac{1}{4} = \frac{13}{4} \text{ gives } \frac{4}{13}$$

3. 0 has no reciprocal.

Check: $\frac{13}{20} \cdot \frac{20}{13} = 1$ and $\frac{13}{4} \cdot \frac{4}{13} = 1$.

Example 2. Evaluate $\frac{3}{14} - \frac{6}{7}$ and $2.1 \div (-0.3)$ using inverses.

1. Subtracting $\frac{6}{7}$ is adding its opposite, and $\frac{6}{7} = \frac{12}{14}$.

$$\frac{3}{14} + \left(-\frac{12}{14}\right) = -\frac{9}{14}$$

2. $-0.3 = -\frac{3}{10}$, so its reciprocal is $-\frac{10}{3}$. Dividing by −0.3 is multiplying by $-\frac{10}{3}$.

$$2.1 \cdot \left(-\frac{10}{3}\right) = -\frac{21}{3} = -7$$

Answer: $-\frac{9}{14}$ and −7. **Check:** −7 · (−0.3) = 2.1.

Example 3. Solve $\frac{5}{6}w - 2 = 13$.

1. The last thing done to w was taking away 2. Add 2 to both sides: −2 + 2 = 0, and adding 0 changes nothing:

$$\frac{5}{6}w = 15$$

2. Multiply both sides by $\frac{6}{5}$, the reciprocal of $\frac{5}{6}$. Their product is 1, and $1 \cdot w = w$.

$$w = \frac{6}{5} \cdot 15 = 18$$

Answer: $w = 18$. **Check:** $\frac{5}{6} \cdot 18 - 2 = 15 - 2 = 13$.

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