# Interpret the parts of an expression

*An expression written for a story tells you about the story before you work anything out. Read it piece by piece.*

## WORDS TO KNOW

**Interpret:** say what each part means in the story, with units.

**Term:** a piece added or subtracted at the top level, with its sign.

**Factor:** a piece that is multiplied. In $5a(a+2)$ the factors are $5$, $a$ and $a+2$.

**Coefficient:** the number factor of a term, like $9$ in $9n$.

**Base and exponent:** in $(1.05)^t$, the exponent $t$ counts the factors of the base $1.05$.

**Parentheses:** a pair stands for one number, however much is inside.

## KEY IDEAS

### 1. The last step decides the shape

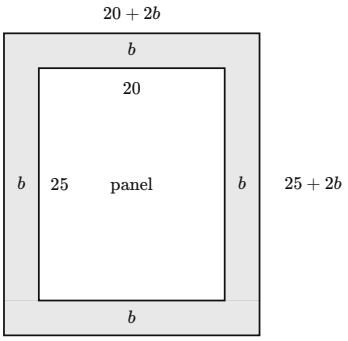
Work it out in the order of operations and look at the last step.

> Last step add or subtract: it is a **sum**, and its parts are **terms**.
>
> Last step multiply: it is a **product**, and its parts are **factors**.
>
> Then read each part the same way, from the top down.

So $8+3k^2$ is a sum of two terms, and $2(m+9)$ is a product of two factors.

### 2. A pair of parentheses is one number

The rest of the expression sees only the value inside each pair of parentheses. A border $b$ wide around a 20 by 25 panel has area $(20+2b)(25+2b)-500$: the whole outer rectangle minus the panel. No multiplying needed.




### 3. A square is never negative

> $u^2 \geq 0$ for every number $u$, and $u^2 = 0$ only when $u = 0$.
>
> So $A+u^2$ is at least $A$, and $A-u^2$ is at most $A$.

### 4. Change one part, and you know what the whole does

> Multiply one factor by $k$: the product is multiplied by $k$.
>
> Raise one term by $d$: the sum rises by $d$.



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## WATCH OUT

- **The meaning comes from the story.** Without one, you can only describe the structure.
- **A formula holds only where the story does.** A count of people is a whole number, though the expression works for fractions.
- **Where the parentheses close decides what they cover.** In $1.07(18h) + 6$ the 6 is added after the tax, so it is untaxed.
- **A letter in two places changes two parts.** In the border area, $b$ is in both factors, so doubling $b$ more than doubles the area.

## WORKED EXAMPLES

**Example 1. A bike costs 18 dollars an hour plus a 6 dollar fee, with 7% tax on the whole bill. For $h$ hours the bill is $1.07(18h + 6)$ dollars. Read the parts, and find the bill for 2 hours.**

1. The last step is multiplying: a product of 1.07 and the sum $18h + 6$.
2. The part in parentheses is the bill before tax: $18h$ for $h$ hours, plus the fee.
3. Read 1.07 as 1 + 0.07: the whole bill plus 7% of it, fee included.
4. For 2 hours: 1.07(36 + 6) = 1.07 × 42 = 44.94.

**Answer:** 44.94 dollars. **Check:** 42 + 0.07 × 42 = 42 + 2.94 = 44.94.

**Example 2. A balance after $t$ years is $1200(1.05)^t$ dollars. Read the parts, and find the balance after 2 years.**

1. The last step is multiplying: a product of 1200 and $(1.05)^t$.
2. 1200 is the deposit, in dollars.
3. Each year the balance is multiplied by 1.05 = 1 + 0.05: all of it, plus 5%.
4. $(1.05)^t$ has one factor of 1.05 for each of the $t$ years.
5. After 2 years: 1.05 × 1.05 = 1.1025, and 1200 × 1.1025 = 1323.

**Answer:** 1323 dollars. **Check:** 1200 + 60 = 1260, then 1260 + 63 = 1323.

**Example 3. Find the smallest value of $(x - 7)^2 + 11$ and the largest value of $17 - (x + 9)^2$.**

1. Use the structure, not trial values. The first is a square, which is at least 0, plus 11. So it is at least 11.
2. It is 11 when the square is 0: $x - 7 = 0$, so $x = 7$.
3. The second is 17 minus a square. Taking away at least 0 leaves at most 17.
4. It is 17 when $x + 9 = 0$, so $x = -9$.

**Answer:** smallest 11 at $x = 7$; largest 17 at $x = -9$.


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