Reasoning with units

Numbers in real problems come with units. The units tell you what to multiply or divide, warn you when a set-up is wrong, and help you decide how many digits to keep.

WORDS TO KNOW

Unit: what a number counts or measures, such as km, h or L.

Cancel: a unit on top and the same unit underneath divide out, like letters.

Exact number: a count or a definition, such as 60 minutes in an hour.

Rate: one unit per another, such as km/h, read "kilometers per hour."

Conversion factor: a fraction like $1000\ \text{m}/1\ \text{km}$ that equals one.

Measured number: a reading from a tool. It is only as good as the tool.

KEY IDEAS

1. Units act like letters

Units multiply, divide and cancel just like letters in algebra. Two quantities can be added or subtracted only when they have the same unit.

$$3\ \text{h} \times \frac{70\ \text{km}}{1\ \text{h}} = 210\ \text{km}$$

Hours cancel, so the answer is a distance.

2. A conversion factor changes the unit, not the amount

Since 1 km = 1000 m, the fraction $\frac{1000\ \text{m}}{1\ \text{km}}$ equals 1. Multiplying by 1 keeps the amount the same.

Put the unit you want to get rid of on the **opposite** side of the bar, so it cancels.

Write the whole chain before any arithmetic. If the unit left over is not the one you want, the set-up is wrong.

3. Keep a sensible number of digits

An answer can be no more accurate than what it was built from.

Keep about as many significant figures as the **measured** number with the fewest.

Exact numbers (counts and unit definitions) never limit the answer. Round once, at the end.

WATCH OUT

- **Upside-down factor.** If a factor is the wrong way up, nothing cancels and the units come out strange, like h^2/km. Flip the factor.
- **Adding different units.** 7 km + 80 m is 7.08 km, never 87.
- **Right units are not a right answer.** A factor like $7\ \text{days}/1\ \text{month}$ has good units but a false value. Every factor must be a true equality.
- **The situation can decide the rounding.** You cannot buy part of a carton, so round up.

Free sample from math.wiki for classroom use. Do not post. math.wiki

WORKED EXAMPLES

Example 1. A car drives at 72 km/h. Write its speed in meters per second.

1. Kilometers are on top, so use the factor with kilometers underneath.
2. Hours are underneath, so use the factor with hours on top.

$$\frac{72\ \text{km}}{1\ \text{h}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{h}}{3600\ \text{s}} = \frac{72{,}000}{3600}\ \frac{\text{m}}{\text{s}}$$

Answer: 20 m/s. **Check:** in one hour, 20 m/s × 3600 s = 72,000 m = 72 km.

Example 2. A car uses 8.5 liters per 100 km. How much fuel does a 265 km trip need?

1. Kilometers must cancel, so the rate goes with kilometers underneath.

$$265\ \text{km} \times \frac{8.5\ \text{L}}{100\ \text{km}} = 22.525\ \text{L}$$

2. The 100 is part of the unit's name, so it is exact. The 8.5 has two significant figures, so keep two.

Answer: about 23 L. **Check:** 22.525 ÷ 265 × 100 = 8.5.

Example 3. A recipe for 24 muffins uses 420 mL of milk. Milk comes in 500 mL cartons. How many cartons do you buy for 30 muffins?

1. Scale factor: $\frac{30\ \text{muffins}}{24\ \text{muffins}} = 1.25$. The units cancel, so it is a plain number.
2. Milk: 420 mL × 1.25 = 525 mL.
3. Cartons: $525\ \text{mL} \times \frac{1\ \text{carton}}{500\ \text{mL}} = 1.05$ cartons. One is too few, so round up.

Answer: 2 cartons.

Free sample from math.wiki for classroom use. Do not post. math.wiki