Writing sets: roster, rule and the empty set

How do you write down exactly which things belong to a set, when there are a few of them, a great many, endlessly many, or none at all?

WORDS TO KNOW

Set: a collection of things. Each thing in it is an **element**.

Ellipsis: three dots, . . ., read "and so on".

Finite: you can count the elements and the count ends.

Whole numbers: 0, 1, 2, 3 and so on. **Integers:** the whole numbers and their negatives.

Roster: a list of a set's elements inside braces, { }, with commas between them.

Empty set: the set with no elements, written ∅ or { }.

Infinite: not finite. However many you count, more are left.

KEY IDEAS

1. A roster names exactly its entries

$\{2, 7, 9\}$ is the set whose elements are 2, 7 and 9, and nothing else.

Order and repeats do not matter: $\{9, 2, 7, 7\}$ is the same set.

2. Dots continue a pattern

The first few entries set up a pattern, and the dots say "keep going the same way". Where the dots sit tells you whether the list ever stops.

$\{5, 10, 15, \ldots, 60\}$ stops at 60: finite.

$\{5, 10, 15, \ldots\}$ never stops: infinite.

$\{\ldots, -15, -10, -5\}$ never stops going down: infinite.

3. A rule in words can name any set

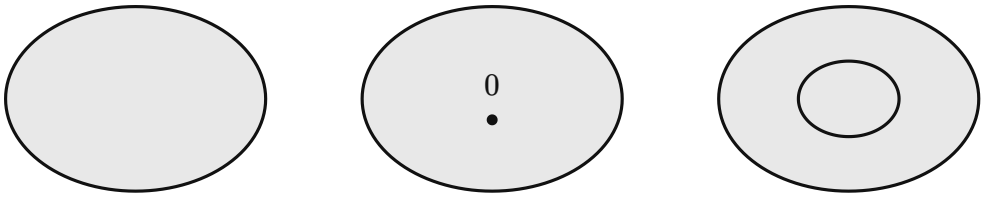
A rule names a set when it gives a clear yes or no for every object. To check a rule against a roster, test it both ways.

The rule must let in every element **and nothing else**.

4. The empty set

Think of a set as a box. ∅ is an empty box. $\{0\}$ holds the number 0, and $\{\varnothing\}$ holds an empty box. Neither of those is empty.

There is only one empty set. Any description that lets nothing in names ∅.



$\varnothing$ — no elements

$\{0\}$ — one element

$\{\varnothing\}$ — one element

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WATCH OUT

- $\varnothing$ **is not the number 0.** The set $\{0\}$ has one element, so it is not empty.
- **The dots need one clear pattern.** $\{1, 2, 4, \dots\}$ could go on 8, 16 (doubling) or 7, 11 (add 1, 2, 3, then 4). If there is any doubt, write the rule in words.
- **Say what kind of number you mean.** "The numbers between 1 and 4" is $\{2, 3\}$ for whole numbers, but infinite if every number counts.

WORKED EXAMPLES

Example 1. Let $A = \{4, 8, 12, \dots, 44\}$ and $B = \{4, 8, 12, \dots\}$. Describe each set. Is 28, 30 or 72 in it?

1. Both start at 4 and go up by 4, so both hold positive multiples of 4.
2. A stops at $44 = 4 \cdot 11$. It is finite with 11 elements.
3. B has nothing after the dots: all positive multiples of 4. It is infinite.
4. $28 = 4 \cdot 7$ is in both. $30 = 4 \cdot 7 + 2$ is in neither. $72 = 4 \cdot 18$ is in B, but it is past 44, so not in A.

Answer: A is finite with 11 elements; B is infinite. 28 is in both, 30 in neither, 72 only in B.

Example 2. Give a rule in words for $\{3, 6, 9, 12, 15\}$.

1. Guess: "the multiples of 3 less than 16".
2. Test the other way. That rule also lets in $0 = 3 \cdot 0$ and -3, which are not in the set.
3. Fix it: "the positive multiples of 3 less than 16". The next one, 18, is too big.

Answer: the positive multiples of 3 less than 16.

Example 3. Write each set as a roster: (a) the whole numbers strictly between 3 and 5, (b) the whole numbers strictly between 3 and 4, (c) the solutions of $x + 1 = x + 6$.

1. (a) "Strictly" leaves out the ends, 3 and 5. Only 4 is left: $\{4\}$, one element.
2. (b) 3 and 4 are next to each other, so no whole number lies between them. The set is $\varnothing$.
3. (c) Subtract x from both sides: $1 = 6$, false for every x. The set is $\varnothing$.

Answer: $\{4\}$, $\varnothing$, $\varnothing$. Sets (b) and (c) look very different, but both have no elements, so they are the same set.

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