Algebra 1 Unit 1 quiz 1: Elements and membership, Writing sets: roster, rule and the empty set (answer key)

8 questions, 20 points

ANSWERS AT A GLANCE

1. $5 \notin E$, $-2 \in E$, $-5 \in E$
2. $\{0, 1, 2, \ldots, 7\}$, with 8 elements
3. $S = \{-6, 9\}$, which has 2 elements.
4. (a) $\{-40, -30, -20, -10, 0, 10, 20, 30, 40\}$, 9 elements. (b) $\{\ldots, -20, -16, -12\}$, infinite. (c) $\{12, 21, 30\}$, 3 elements. (d) $\varnothing$, 0 elements.
5. (a) A, B and C have 2 elements each; D has 1. (b) $A = B = C$, and D is equal to none of them.
6. (a) $\varnothing$. (b) The set of all real numbers (every real number is a solution), which is infinite. (c) $\{0\}$.
7. (a) $k = 4$, $k = \sqrt{17}$ or $k = -\sqrt{17}$. (b) $S = \{17, 16\}$, which has 2 elements.
8. No. Grace's rule names exactly $\{1, 3, 5, 7, 9\}$: each of these is an odd whole number less than 10, and the only other whole numbers less than 10 are 0, 2, 4, 6, 8, which are even. 0 is a whole number but it is even, so the rule does not let it in.

WORKED SOLUTIONS

1. Let E be the set of solutions of the equation $x^2 + 7x = -10$. Decide whether each of 5, −2 and −5 is an element of E, and write each answer with $\in$ or $\notin$. (2 points)

ANSWER

$5 \notin E$, $-2 \in E$, $-5 \in E$

SOLUTION

A number is an element of E exactly when putting it in for x makes the equation true, that is, when $x^2 + 7x$ comes out to −10.

- $x = 5$: $5^2 + 7 \cdot 5 = 60$, not −10, so $5 \notin E$.
- $x = -2$: $(-2)^2 + 7 \cdot (-2) = -10$, so $-2 \in E$.
- $x = -5$: $(-5)^2 + 7 \cdot (-5) = -10$, so $-5 \in E$.

2. Write as a roster the set of whole numbers n for which $3n + 2$ is less than 24. How many elements does it have? (2 points)

ANSWER

$\{0, 1, 2, \ldots, 7\}$, with 8 elements

SOLUTION

The value of $3n + 2$ grows as n grows. At $n = 0$ it is 2, less than 24, so 0 is in the set (0 is a whole number). At $n = 7$ it is 23, still less than 24; at $n = 8$ it is 26, which is not, and larger n give larger values.

So the set is $\{0, 1, 2, \ldots, 7\}$, the whole numbers from 0 to 7, with 8 elements.

SCORING

1 point for the roster from 0 to 7 (including 0); 1 point for 8 elements.

3. Let $x = -3$, and let $S = \{x - 3,\ 2x,\ x^2,\ 9\}$. Write S with each element listed once, and say how many elements it has. (2 points)

ANSWER

$S = \{-6, 9\}$, which has 2 elements.

SOLUTION

Work out each listed object at $x = -3$:

$$x - 3 = -6, \qquad 2x = -6, \qquad x^2 = 9, \qquad 9 = 9.$$

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Repeats collapse, so $S = \{-6, -6, 9, 9\} = \{-6, 9\}$. The two values are different, so S has 2 elements, even though four things were written.

4. Write each set as a roster, using an ellipsis where one is needed. Then give its number of elements, or say that it is infinite. (3 points)

(a) The multiples of 10 strictly between −50 and 50.

(b) The multiples of 4 that are less than −10.

(c) The two-digit whole numbers whose digits add up to 3.

(d) The whole numbers n for which $n + 7$ is less than 7.

ANSWER

(a) $\{-40, -30, -20, -10, 0, 10, 20, 30, 40\}$, 9 elements. (b) $\{\ldots, -20, -16, -12\}$, infinite. (c) $\{12, 21, 30\}$, 3 elements. (d) $\varnothing$, 0 elements.

SOLUTION

(a) The set is $\{-40, -30, -20, -10, 0, 10, 20, 30, 40\}$; $0 = 10 \cdot 0$ is a multiple of 10, and −50 and 50 are left out because the ends are excluded. Finite, with 9 elements.

(b) The multiples of 4 near −10 are −8 and −12; −8 is not less than −10, so the largest element is −12. Below it come −16, −20 and so on downward forever, so the dots go at the front: $\{\ldots, -20, -16, -12\}$. Infinite.

(c) A two-digit number has a first digit of at least 1. First digit 1 needs second digit 2, first digit 2 needs 1, first digit 3 needs 0, and a first digit of 4 or more is already too big. The set is $\{12, 21, 30\}$, finite with 3 elements.

(d) Subtracting 7 from both sides, $n + 7$ is less than 7 exactly when n is less than 0, and no whole number is less than 0. The set is $\varnothing$. It is finite, with 0 elements.

SCORING

1 point for (a) finite with 9 elements (including 0); 1 point for (b) infinite with the dots at the front ending at −12, and (c) 3 elements; 1 point for (d) $\varnothing$, finite with 0 elements.

5. Let $A = \{2, \{2\}\}$, $B = \{\{2\}, 2, 2\}$, $C = \{2, \{2, 2\}\}$ and $D = \{\{2\}, \{2\}\}$. (3 points)

(a) How many elements does each set have?

(b) Which of the four sets are equal to each other? Explain using the rule for equal sets.

ANSWER

(a) A, B and C have 2 elements each; D has 1. (b) $A = B = C$, and D is equal to none of them.

SOLUTION

(a) A has the elements 2 and $\{2\}$, which are different objects: 2 elements.

B lists $\{2\}$, 2 and 2 again. The repeat adds nothing, so its elements are $\{2\}$ and 2: 2 elements.

C lists 2 and $\{2, 2\}$. The inner set $\{2, 2\}$ is the set $\{2\}$, since repeats collapse inside it too. So the elements of C are 2 and $\{2\}$: 2 elements.

D lists $\{2\}$ twice, so its only element is $\{2\}$: 1 element.

(b) Two sets are equal exactly when they have the same elements. A, B and C each have exactly the elements 2 and $\{2\}$, so $A = B = C$. But $2 \in A$ and $2 \notin D$, so D is not equal to A, and so not to B or C either.

SCORING

1 point for the four counts in (a), including C having 2 elements and D having 1. 1 point for $A = B = C$, with $\{2, 2\} = \{2\}$ given as the reason C belongs. 1 point for showing D differs, by naming an element (2)

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in one set and not the other.

6. Write the solution set of each equation. Use a roster or $\varnothing$ where you can; otherwise describe the set in words. (3 points)

(a) $4(x - 7) = 4x - 4$

(b) $3(x + 7) = 3x + 21$

(c) $9(x + 7) = 63$

ANSWER

(a) $\varnothing$. (b) The set of all real numbers (every real number is a solution), which is infinite. (c) $\{0\}$.

SOLUTION

(a) Distributing gives $4x - 28 = 4x - 4$. Subtracting $4x$ from both sides leaves $-28 = -4$, which is false whatever x is. No number is a solution: the set is $\varnothing$.

(b) Distributing gives $3x + 21 = 3x + 21$, the same on both sides, so the equation is true for every x. The solution set is the set of all real numbers. It is infinite and has no next member, so it is described in words, not by a roster.

(c) Distributing gives $9x + 63 = 63$. Subtracting 63 from both sides gives $9x = 0$, so $x = 0$, and $9(0 + 7) = 63$ checks. The set is $\{0\}$, with one element; it is not empty.

SCORING

1 point for each part: (a) $\varnothing$; (b) all real numbers, in words; (c) $\{0\}$, not $\varnothing$.

7. Let $S = \{k + 13,\ k^2,\ 2k + 9\}$, where k is a real number. (3 points)

(a) Find every value of k for which $17 \in S$.

(b) One of those values is a whole number. For that value of k, write S with each element listed once and say how many elements it has.

ANSWER

(a) $k = 4$, $k = \sqrt{17}$ or $k = -\sqrt{17}$. (b) $S = \{17, 16\}$, which has 2 elements.

SOLUTION

(a) $17 \in S$ means 17 equals at least one of the three listed objects. Take each in turn.

- $k + 13 = 17$ gives $k = 4$.
- $k^2 = 17$ gives $k = \sqrt{17}$ or $k = -\sqrt{17}$.
- $2k + 9 = 17$ gives $2k = 8$, so $k = 4$ again.

So $17 \in S$ exactly when $k = 4$, $k = \sqrt{17}$ or $k = -\sqrt{17}$.

(b) At $k = 4$ the listed objects are $4 + 13 = 17$, $4^2 = 16$ and $2 \cdot 4 + 9 = 17$. The repeat collapses, so $S = \{17, 16\}$, with 2 elements.

SCORING

1 point for setting 17 equal to each listed object in turn. 1 point for all three values $k = 4$, $\sqrt{17}$, $-\sqrt{17}$ (no value listed twice is fine; missing $-\sqrt{17}$ loses this point). 1 point for $S = \{17, 16\}$ with 2 elements.

8. Grace describes the set $\{1, 3, 5, 7, 9\}$ as "the odd whole numbers less than 10". Hannah says the word "whole" should be "positive", because otherwise the rule lets in 0. Is Hannah right that Grace's rule lets in a number that is not in the set? Explain by testing Grace's rule both ways. (2 points)

ANSWER

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No. Grace's rule names exactly $\{1, 3, 5, 7, 9\}$: each of these is an odd whole number less than 10, and the only other whole numbers less than 10 are 0, 2, 4, 6, 8, which are even. 0 is a whole number but it is even, so the rule does not let it in.

SOLUTION

A rule names a set exactly when it lets in every element and nothing else, so test it both ways.

Every element of $\{1, 3, 5, 7, 9\}$ is odd, is a whole number and is less than 10, so the rule lets in every element.

The whole numbers less than 10 are 0, 1, 2, ..., 9. Of these, 0, 2, 4, 6, 8 are even, so the rule turns them away, and what is left is exactly 1, 3, 5, 7, 9. Negative numbers are not whole numbers, so they are turned away too. The number 0 is a whole number, but $0 = 2 \cdot 0$ is even, so it is not let in.

So Hannah is wrong: Grace's rule is already correct. ("The positive odd numbers less than 10" is also a correct rule, but the change is not needed.)

SCORING

1 point for no, with the reason that 0 is even; 1 point for testing the other way: every listed element meets the rule, and no other whole number less than 10 does.

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