Algebra 1 Unit 1 quiz 2: Commutative, associative and distributive properties, Identity and inverse elements, Closure (answer key)

8 questions, 21 points

ANSWERS AT A GLANCE

1. $h = 3$ and $k = 5$
2. $6x - 33$
3. (a) $-\frac{4}{3}$ (b) $-\frac{4}{3}$; the two orders give the same number.
4. The swap gives $(-b) + a$, which is still $a - b$, not $b - a$. For example, 9 − 4 = 5 but 4 − 9 = −5.
5. (a) $-\frac{2}{9}n + 11 = 9$ (b) $n = 9$
6. $ab = 84$, rational; $a \div b = \frac{4}{3}$, rational; $a - b = \sqrt{7}$, irrational
7. (a) $x = 3$ (b) At $x = 0$: $0 \cdot 5 = 0 = 11 \cdot 0$, but $2 \cdot 0 + 5 = 5 \neq 11$. (c) Undoing the step means multiplying by $\frac{1}{x}$, and at $x = 0$ there is no reciprocal, so the new equation gains the solution $x = 0$.
8. See the proof below.

WORKED SOLUTIONS

1. Find the numbers h and k that make $-5(hx - 1) = -15x + k$ true for every real number x. (2 points)

> **ANSWER**
> $h = 3$ and $k = 5$
>
> **SOLUTION**
> By the distributive property, $-5(hx - 1) = -5hx + (-5) \cdot (-1)$.
>
> The x terms must match: $-5h = -15$, so $h = -15 \div (-5) = 3$.
>
> The numbers must match: $k = -5 \cdot (-1) = 5$.
>
> **SCORING**
> 1 point for $h = 3$; 1 point for $k = 5$.

2. Hannah simplified $3 + 6(x - 6)$ by writing

$$3 + 6(x - 6) = 9(x - 6) = 9x - 54$$

Explain the mistake, and give the correct simplified expression. (2 points)

> **ANSWER**
> $6x - 33$
>
> **SOLUTION**
> The factor 6 multiplies only the parentheses; the 3 is added afterwards, so it cannot be joined to the 6 first. No property turns $3 + 6(x - 6)$ into $(3 + 6)(x - 6)$: the distributive property hands a factor to each term of a sum, it does not let a term be added to a factor.
>
> Correctly: $3 + 6(x - 6) = 3 + 6x - 36$ (distributive property) $= 6x + 3 - 36$ (commutative and associative properties of addition) $= 6x - 33$.
>
> **SCORING**
> 1 point for saying the 3 was wrongly combined with the factor 6 before multiplying; 1 point for $6x - 33$.

3. Let $x = 0.75$. (2 points)

(a) Find the multiplicative inverse of the additive inverse of x.

(b) Find the additive inverse of the multiplicative inverse of x.

Give both answers as fractions, and say whether the two orders give the same number.

> **ANSWER**

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(a) $-\frac{4}{3}$ (b) $-\frac{4}{3}$; the two orders give the same number.

SOLUTION

As a fraction, $0.75 = \frac{3}{4}$.

(a) The additive inverse of $\frac{3}{4}$ is $-\frac{3}{4}$. Its multiplicative inverse is the fraction turned over with its sign kept, $-\frac{4}{3}$. Check: $\left(-\frac{3}{4}\right)\left(-\frac{4}{3}\right) = 1$.

(b) The multiplicative inverse of $\frac{3}{4}$ is $\frac{4}{3}$, and its additive inverse is $-\frac{4}{3}$.

The two orders agree: $(-x)\left(-\frac{1}{x}\right) = x \cdot \frac{1}{x} = 1$, so $-\frac{1}{x}$ is the reciprocal of $-x$.

SCORING

1 point for (a), $-\frac{4}{3}$; 1 point for (b), $-\frac{4}{3}$, with the statement that the two agree.

4. Mei argues: "Subtraction is commutative after all. We can write $a - b$ as $a + (-b)$, and addition is commutative."

Carry out her swap on $a + (-b)$. Is the result $b - a$? Explain where her argument goes wrong, and give numbers showing that $a - b$ and $b - a$ can differ. (2 points)

ANSWER

The swap gives $(-b) + a$, which is still $a - b$, not $b - a$. For example, 9 − 4 = 5 but 4 − 9 = −5.

SOLUTION

By the commutative property of addition, $a + (-b) = (-b) + a$. The term $-b$ moves with its sign attached, so $(-b) + a$ is the same number as $a - b$. To get $b - a$ the signs would have to change too: $b - a = b + (-a)$, which has b positive and a negative.

So the swap is correct but proves only that $a - b = -b + a$; it says nothing about $b - a$.

Counterexample: with $a = 9$ and $b = 4$, 9 + (−4) = 5 and (−4) + 9 = 5, but 4 − 9 = −5.

SCORING

1 point for $(-b) + a$ with the explanation that it is still $a - b$ (signs move with their terms); 1 point for a correct counterexample.

5. Ethan thinks of a number, multiplies it by $-\frac{2}{9}$, then adds 11, and ends with 9. (3 points)

(a) Write an equation for Ethan's number n.

(b) Solve it, naming the inverse you use at each step.

ANSWER

(a) $-\frac{2}{9}n + 11 = 9$ (b) $n = 9$

SOLUTION

(a) $-\frac{2}{9}n + 11 = 9$.

(b) Undo the last step first. Add −11, the additive inverse of 11, to both sides; 11 + (−11) = 0 and adding 0 changes nothing:

$$-\frac{2}{9}n = 9 - 11 = -2.$$

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Multiply both sides by $-\frac{9}{2}$, the multiplicative inverse of $-\frac{2}{9}$; their product is 1 and $1 \cdot n = n$:

$$n = -\frac{9}{2} \cdot (-2) = 9.$$

Check: $-\frac{2}{9} \cdot 9 + 11 = -2 + 11 = 9$.

SCORING

1 point for the equation; 1 point for adding −11 to both sides to reach $-\frac{2}{9}n = -2$; 1 point for multiplying by $-\frac{9}{2}$ to get $n = 9$.

6. Let $a = 4\sqrt{7}$ and $b = 3\sqrt{7}$, two irrational numbers. Find ab, $a \div b$ and $a - b$ in simplest form, and say whether each is rational or irrational. (3 points)

ANSWER

$ab = 84$, rational; $a \div b = \frac{4}{3}$, rational; $a - b = \sqrt{7}$, irrational

SOLUTION

$ab = 4 \times 3 \times \sqrt{7} \times \sqrt{7} = 4 \times 3 \times 7 = 84$, rational.

$a \div b = \frac{4\sqrt{7}}{3\sqrt{7}} = \frac{4}{3}$, rational, since the nonzero factor $\sqrt{7}$ cancels.

$a - b = (4 - 3)\sqrt{7} = \sqrt{7}$ by the distributive property. This is a nonzero rational number times the irrational $\sqrt{7}$, so it is irrational.

So with these two irrational numbers, the product and the quotient leave the irrational numbers and the difference does not.

SCORING

1 point for $ab = 84$, rational; 1 point for $a \div b = \frac{4}{3}$, rational; 1 point for $a - b = \sqrt{7}$, irrational, with a reason.

7. Kofi multiplies both sides of $2x + 5 = 11$ by x and gets $x(2x + 5) = 11x$. (3 points)

(a) Solve $2x + 5 = 11$.

(b) Show that $x = 0$ makes $x(2x + 5) = 11x$ true but $2x + 5 = 11$ false.

(c) Explain, using reciprocals, why Kofi's step cannot be undone for every x.

ANSWER

(a) $x = 3$ (b) At $x = 0$: $0 \cdot 5 = 0 = 11 \cdot 0$, but $2 \cdot 0 + 5 = 5 \neq 11$. (c) Undoing the step means multiplying by $\frac{1}{x}$, and at $x = 0$ there is no reciprocal, so the new equation gains the solution $x = 0$.

SOLUTION

(a) Add −5 to both sides: $2x = 6$. Multiply by $\frac{1}{2}$: $x = 3$.

(b) At $x = 0$ the new equation reads $0 \cdot (0 + 5) = 11 \cdot 0$, that is $0 = 0$, which is true. The original reads $0 + 5 = 11$, which is false.

(c) A step on both sides keeps the solutions exactly when it can be reversed. To reverse multiplying by x you would multiply both sides by $\frac{1}{x}$, the reciprocal of x, which exists only when $x \neq 0$. At $x = 0$ the step sends both sides to 0 and cannot be undone, so the new equation picks up the extra solution $x = 0$. Its solutions are 3 and 0; only 3 solves the original.

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SCORING

1 point for $x = 3$; 1 point for both substitutions of $x = 0$; 1 point for saying the reverse step needs $\frac{1}{x}$, which does not exist at $x = 0$ because 0 has no reciprocal.

8. Show that $(a + b) + (c + d) = (a + c) + (b + d)$ for all real numbers a, b, c and d, naming the property used at each step. Change only one thing (an order or a grouping) at each step. (4 points)

ANSWER

The statement is true for all real numbers; see the proof.

SOLUTION

Expression	Reason
$(a + b) + (c + d)$	
$= a + \big(b + (c + d)\big)$	associative property of addition
$= a + \big((b + c) + d\big)$	associative property of addition
$= a + \big((c + b) + d\big)$	commutative property of addition
$= a + \big(c + (b + d)\big)$	associative property of addition
$= (a + c) + (b + d)$	associative property of addition

(In the first step the sum $c + d$ is treated as one number.) Other correct orders of steps earn full credit.

SCORING

1 point for regrouping so that b and c stand together, with the associative property named; 1 point for swapping b and c by the commutative property of addition; 1 point for regrouping to $(a + c) + (b + d)$ with the associative property named; 1 point for every step changing only one thing, each with its correct reason.

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