Algebra 1 Unit 1 quiz 3: Interpreting the parts of an expression, Units, quantities and a sensible level of accuracy, Estimating a huge quantity from small facts (answer key)

8 questions, 21 points

ANSWERS AT A GLANCE

1. 21 meters per minute
2. 525 cm
3. (a) The sum in parentheses, 29 km/h, is how fast the gap between them closes; $(13+16)t$ is the distance, in km, by which the gap has shrunk after t hours. (b) 29 km (c) 2 hours
4. (a) 160 trucks (b) 240 trucks
5. (a) 4, at $x=0$. (b) No: the quotient $\dfrac{12}{x^2+2}$ is always positive, so something positive is always taken from 10 and the value is always less than 10. (c) $x=\sqrt{2}$ and $x=-\sqrt{2}$
6. (a) Mateo: 10^5, David: 10^6 (b) 5 times; yes, they agree to an order of magnitude.
7. About 140 days
8. 6 mL

WORKED SOLUTIONS

1. A conveyor belt at a grocery checkout moves at 35 centimeters per second. Write its speed in meters per minute. (2 points)

ANSWER

21 meters per minute

SOLUTION

Centimeters are on top and must become meters (1 m = 100 cm); seconds are underneath and must become minutes:

$$\frac{35\text{ cm}}{1\text{ s}}\times\frac{60\text{ s}}{1\text{ min}}\times\frac{1\text{ m}}{100\text{ cm}}=\frac{2100}{100}\ \frac{\text{m}}{\text{min}}=21\text{ m/min}$$

SCORING

1 point for a chain in which seconds and centimeters both cancel; 1 point for 21 meters per minute.

2. A roll holds 7 m of ribbon. Hannah cuts 7 pieces, each 25 cm long. How many centimeters of ribbon are left on the roll? (2 points)

ANSWER

525 cm

SOLUTION

Subtraction needs one unit, so write the roll in centimeters: $7\text{ m}\times\dfrac{100\text{ cm}}{1\text{ m}}=700\text{ cm}$.

The pieces use 7 × 25 cm = 175 cm.

Left: 700 cm − 175 cm = 525 cm.

SCORING

1 point for writing the roll as 700 cm (or the pieces in meters); 1 point for 525 cm.

3. Hannah and Carlos cycle toward each other along a straight road, starting 58 km apart. Hannah rides at 13 km/h and Carlos at 16 km/h. After t hours the distance between them is

$$58-(13+16)t$$

kilometers. (3 points)

(a) What does the sum in parentheses, (13 + 16), stand for, with units, and what does $(13+16)t$ measure?

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(b) How far apart are they after 1 hour?

(c) After how many hours do they meet?

ANSWER

(a) The sum in parentheses, 29 km/h, is how fast the gap between them closes; $(13 + 16)t$ is the distance, in km, by which the gap has shrunk after t hours. (b) 29 km (c) 2 hours

SOLUTION

(a) In one hour Hannah covers 13 km and Carlos covers 16 km, and both distances come off the gap. So the sum in parentheses, 13 + 16 = 29 km/h, is the speed at which the gap closes. Multiplied by the time t in hours, $(13 + 16)t$ is the total distance the two have ridden, which is how much the gap has shrunk, in km. The whole expression is the starting gap minus that.

(b) 58 − 29 × 1 = 29 km.

(c) They meet when the gap is 0: $29t = 58$, so $t = 58 \div 29 = 2$ hours.

SCORING

1 point for reading the sum in parentheses as the closing speed in km/h and the product as distance closed; 1 point for 29 km; 1 point for 2 hours.

4. Owen estimates how many garbage trucks a city needs. Assumptions: the city has about 80,000 homes, each home puts out about 2 bags of trash a week, and one truck can collect about 1,000 bags a week. (3 points)

(a) Find the estimate, with the units shown.

(b) A classmate argues that there are 3 times as many homes as Owen assumed, and that one truck can collect 2 times as many bags. Without redoing the whole chain, find the new estimate.

ANSWER

(a) 160 trucks (b) 240 trucks

SOLUTION

(a)

$$80000 \text{ homes} \times \frac{2 \text{ bags per week}}{1 \text{ home}} \times \frac{1 \text{ truck}}{1000 \text{ bags per week}} = 160 \text{ trucks}$$

Homes cancel and bags per week cancel, leaving trucks.

(b) The homes factor is multiplied by 3 and the bags-per-truck factor, which divides, is multiplied by 2. So the estimate is multiplied by $\frac{3}{2}$: $160 \times \frac{3}{2} = 240$ trucks.

SCORING

1 point for a chain whose units cancel to trucks and the value 160; 1 point for seeing that the estimate is multiplied by $\frac{3}{2}$ (the truck factor divides); 1 point for 240 trucks.

5. Answer from the structure of $10 - \frac{12}{x^2 + 2}$, not from a table of values. (3 points)

(a) What is the smallest value of the expression, and for which x does it happen?

(b) Can the expression ever equal 10? Explain.

(c) Find every x for which it equals 7.

ANSWER

(a) 4, at $x = 0$. (b) No: the quotient $\frac{12}{x^2 + 2}$ is always positive, so something positive is always taken from 10 and the value is always less than 10. (c) $x = \sqrt{2}$ and $x = -\sqrt{2}$

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SOLUTION

(a) The last step is subtracting: this is 10 minus the quotient $\dfrac{12}{x^2+2}$. The denominator x^2+2 is at least 2, since $x^2 \geq 0$. Dividing 12 by a larger positive number gives a smaller result, so the quotient is at most $12 \div 2 = 6$, reached at $x = 0$. Taking away at most 6 leaves at least $10 - 6 = 4$, so the smallest value is 4, at $x = 0$.

(b) No. The quotient is 12 divided by a positive number, so it is positive and never 0. The expression is 10 minus a positive number, so it is always less than 10, however large x is.

(c) The expression is 7 when the quotient taken away is $10 - 7 = 3$. Then 12 divided by the denominator is 3, so the denominator is $12 \div 3 = 4$: $x^2 + 2 = 4$ and $x^2 = 2$. So $x = \sqrt{2}$ or $x = -\sqrt{2}$.

SCORING

1 point for the smallest value 4 at $x = 0$, with the reason that the quotient is at most 6; 1 point for "no" in (b) with the reason that the quotient is always positive; 1 point for both $x = \sqrt{2}$ and $x = -\sqrt{2}$.

6. For the number of bricks in a school building, Mateo estimates 800,000 and David estimates 4,000,000. (3 points)

(a) Give the order of magnitude of each estimate.

(b) How many times the smaller estimate is the larger? Do the two estimates agree to an order of magnitude?

ANSWER

(a) Mateo: 10^5, David: 10^6 (b) 5 times; yes, they agree to an order of magnitude.

SOLUTION

(a) 800,000 is 8×10^5, order of magnitude 10^5. 4,000,000 is 4×10^6, order of magnitude 10^6.

(b) The larger estimate, 4,000,000, divided by the smaller, 800,000, is 5. This is less than 10, so they agree to an order of magnitude, even though their powers of ten differ.

SCORING

1 point for both orders of magnitude in part (a); 1 point for the ratio 5; 1 point for the verdict (yes, they agree to an order of magnitude) with the reason, comparing the ratio with 10.

7. Hannah plans to count aloud from 1 to 1 000 000. Assume each number takes about 2 seconds to say, and that Hannah counts for 4 hours a day. Estimate how many days it takes, to two significant figures, showing the units cancel. (2 points)

ANSWER

About 140 days

SOLUTION

$$10^6 \text{ numbers} \times \frac{2 \text{ s}}{1 \text{ number}} \times \frac{1 \text{ h}}{3600 \text{ s}} \times \frac{1 \text{ day}}{4 \text{ h}}$$

Numbers, seconds and hours cancel, leaving days:

$$\frac{2 \times 10^6}{3600 \times 4} = \frac{2 \times 10^6}{14400} \approx 140 \text{ days}$$

8. A doctor prescribes 10 milligrams of a medicine per kilogram of body mass. The patient's mass is 24 kg, and the liquid medicine contains 200 mg in every 5 mL. Write one chain of factors that turns the patient's mass into a dose in milliliters, and find the dose. (3 points)

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ANSWER

6 mL

SOLUTION

Start from the mass. Kilograms cancel against the prescription, then milligrams against the strength of the liquid, which must have milligrams underneath:

$$24\ \text{kg} \times \frac{10\ \text{mg}}{1\ \text{kg}} \times \frac{5\ \text{mL}}{200\ \text{mg}}$$

The first factor gives $24 \times 10 = 240$ mg, and $240 \times 5 \div 200 = 6$. The dose is 6 mL.

SCORING

1 point for a chain whose units cancel to milliliters (the strength written with milligrams underneath); 1 point for 240 mg; 1 point for 6 mL.

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