# Algebra 1 Unit 1 worksheet: Commutative, associative and distributive properties (answer key)

**ANSWERS AT A GLANCE**

1. (a) $5(x+3)$ (b) $-2+(y+7)$ (c) $4m-4\cdot 6$ (d) $9(k+2)$ (e) $c+(a+b)$, or $(b+a)+c$
2. (a) 13 · 5 + 13 · 2 (b) 13(5 + 2) (c) the distributive property (d) 91 dollars
3. (a) The terms were moved without their signs. (b) $38-x$ (c) The original is 36 and Hannah's answer is 22.
4. $-4(3x-5)$
5. 42 · 62 = 2604
6. (1) associative property of multiplication (2) associative property of multiplication (3) commutative property of multiplication (4) associative property of multiplication (5) associative property of multiplication (6) arithmetic. Both expressions equal −24.
7. See the proof below.

**WORKED SOLUTIONS**

**1.** Complete each statement so that it is an example of the property named. Do not simplify.

(a) Commutative property of multiplication: $(x+3)\cdot 5 =$ _____

(b) Associative property of addition: $(-2+y)+7 =$ _____

(c) Distributive property: $4(m-6) =$ _____

(d) Distributive property, read right to left: $9k+9\cdot 2 =$ _____

(e) Commutative property of addition: $(a+b)+c =$ _____ (two different answers are possible; give one)

**ANSWER**

(a) $5(x+3)$ (b) $-2+(y+7)$ (c) $4m-4\cdot 6$ (d) $9(k+2)$ (e) $c+(a+b)$, or $(b+a)+c$

**SOLUTION**

(a) The two factors $(x+3)$ and 5 trade places: $5(x+3)$. Nothing is multiplied out.

(b) The order $-2$, $y$, 7 stays the same and only the parentheses move: $-2+(y+7)$.

(c) The factor 4 is handed to each term: $4m-4\cdot 6$ (which is $4m-24$).

(d) Both terms share the factor 9, so it comes out in front: $9(k+2)$.

(e) The commutative property swaps two terms of one sum. Either swap the sum $a+b$ with $c$, giving $c+(a+b)$, or swap $a$ and $b$ inside the parentheses, giving $(b+a)+c$. Writing $a+(b+c)$ would be the associative property instead.

**2.** Hannah earns 13 dollars an hour at a bike shop and works 5 hours on Saturday and 2 hours on Sunday.

(a) Write an expression for the weekend's pay that finds each day's pay first.

(b) Write a different expression for the same pay that finds the total number of hours first.

(c) Which property says the two expressions are equal?

(d) Find the weekend's pay.

**ANSWER**

(a) 13 · 5 + 13 · 2 (b) 13(5 + 2) (c) the distributive property (d) 91 dollars

**SOLUTION**

(a) Saturday pays 13 · 5 dollars and Sunday pays 13 · 2 dollars, so the pay is 13 · 5 + 13 · 2.

(b) The total time is 5 + 2 hours at 13 dollars each: 13(5 + 2).


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(c) $13(5+2) = 13 \cdot 5 + 13 \cdot 2$ is the distributive property.

(d) $13(5+2) = 13 \cdot 7 = 91$, and the other way $65 + 26 = 91$. The pay is 91 dollars.

**3.** Hannah simplified $29 - x + 9$ like this:

$$29 - x + 9 = 29 - 9 + x = 20 + x$$

(a) Explain what went wrong.

(b) Simplify $29 - x + 9$ correctly.

(c) Show that Hannah's answer and the original expression have different values when $x = 2$.

**ANSWER**

(a) The terms were moved without their signs. (b) $38 - x$ (c) The original is 36 and Hannah's answer is 22.

**SOLUTION**

(a) Subtraction is not commutative, so terms cannot simply trade places. Writing the expression as $29 + (-x) + 9$ makes it a sum, and then each term moves with its sign attached: the $-$ belongs to $x$, not to 9. Hannah gave the minus sign to 9 and a plus sign to $x$.

(b) $29 - x + 9 = 29 + (-x) + 9 = 29 + 9 + (-x) = 38 - x$, by the commutative property of addition.

(c) At $x = 2$: 29 − 2 + 9 = 36, but 20 + 2 = 22. The values differ, so the two expressions are not equal for every $x$.

**4.** Factor $-12x + 20$ so that the factor outside the parentheses is negative and as large in size as possible. Check your answer by expanding it.

**ANSWER**

$-4(3x - 5)$

**SOLUTION**

The largest whole number dividing both 12 and 20 is 4, so take −4 outside. Then $-12x = -4 \cdot 3x$ and $20 = -4 \cdot (-5)$, and the distributive property read right to left gives

$$-12x + 20 = -4\big(3x + (-5)\big) = -4(3x - 5)$$

Check: $-4 \cdot 3x = -12x$ and $-4 \cdot (-5) = 20$. Note the sign inside the parentheses: taking out a negative factor turns + 20 into −5.

**5.** Write $42 = 40 + 2$ and $62 = 60 + 2$. Use the identity $(a + b)(c + d) = ac + ad + bc + bd$ to work out $42 \cdot 62$ without a calculator, showing the four products.

**ANSWER**

$42 \cdot 62 = 2604$

**SOLUTION**

$$42 \cdot 62 = (40 + 2)(60 + 2) = 40 \cdot 60 + 40 \cdot 2 + 2 \cdot 60 + 2 \cdot 2$$

$$= 2400 + 80 + 120 + 4 = 2604$$

Each part of 42 is multiplied by each part of 62: four products in all.

**6.** Here is why $(3x)(4y)$ may be written $12xy$. Give the property or arithmetic behind each numbered step.


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| Expression | Step |
|---|---|
| $(3x)(4y)$ | |
| $= 3\big(x(4y)\big)$ | (1) |
| $= 3\big((x \cdot 4)y\big)$ | (2) |
| $= 3\big((4x)y\big)$ | (3) |
| $= 3\big(4(xy)\big)$ | (4) |
| $= (3 \cdot 4)(xy)$ | (5) |
| $= 12xy$ | (6) |

Then check that the first and last expressions agree when $x = 2$ and $y = -1$.

**ANSWER**

(1) associative property of multiplication (2) associative property of multiplication (3) commutative property of multiplication (4) associative property of multiplication (5) associative property of multiplication (6) arithmetic. Both expressions equal $-24$.

**SOLUTION**

(1) The factors 3, $x$, $4y$ keep their order; only the parentheses move: associative property of multiplication.

(2) Inside, $x(4y)$ becomes $(x \cdot 4)y$, again only a change of parentheses: associative property of multiplication.

(3) $x \cdot 4$ becomes $4x$, a change of order: commutative property of multiplication.

(4) $(4x)y$ becomes $4(xy)$: associative property of multiplication.

(5) $3\big(4(xy)\big)$ becomes $(3 \cdot 4)(xy)$: associative property of multiplication.

(6) $3 \cdot 4 = 12$: arithmetic.

Check: $(3 \cdot 2)\big(4 \cdot (-1)\big) = 6 \cdot (-4) = -24$, and $12 \cdot 2 \cdot (-1) = -24$.

**7.** Define a new operation on real numbers by $a * b = ab + a + b$.

(a) Find $2 * (-3)$.

(b) Show that $*$ is commutative: $a * b = b * a$ for all real numbers $a$ and $b$. Name the property used at each step.

(c) Show that $*$ is associative: $(a * b) * c = a * (b * c)$ for all real numbers $a$, $b$ and $c$.

**ANSWER**

(a) $-7$ (b) and (c): both sides are equal for all real numbers; see the solution.

**SOLUTION**

(a) $2 * (-3) = 2(-3) + 2 + (-3) = -6 + 2 - 3 = -7$.

(b)

| Expression | Reason |
|---|---|
| $a * b = ab + a + b$ | definition of $*$ |
| $= ba + a + b$ | commutative property of multiplication |
| $= ba + (a + b)$ | associative property of addition |
| $= ba + (b + a)$ | commutative property of addition |
| $= (ba + b) + a$ | associative property of addition |
| $= b * a$ | definition of $*$ |

(c) Work out each side with the distributive property.


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$(a * b) * c = (ab + a + b)c + (ab + a + b) + c = abc + ac + bc + ab + a + b + c$, by the distributive property (with $(ab)c = abc$ by the associative property of multiplication).

$a * (b * c) = a(bc + b + c) + a + (bc + b + c) = abc + ab + ac + a + bc + b + c$, by the distributive property.

Both are sums of the same seven terms $abc$, $ab$, $ac$, $bc$, $a$, $b$, $c$. By the commutative and associative properties of addition a sum has the same total in any order and grouping, so $(a * b) * c = a * (b * c)$.


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