Algebra 1 Unit 1 worksheet: Elements and membership (answer key)

ANSWERS AT A GLANCE

1. $4 \notin B, 7 \in B, 5 \in B, 0 \notin B$
2. (a), (c) and (e) give sets. (b) and (d) do not.
3. (a) 3 times. (b) 1 time. (c) No: the set shows only which numbers came up, not how many times each one did.
4. $68 \in G$ and $71 \notin G$
5. $x = -4$, and the set is $\{-25\}$.
6. (a) The elements are $\{1, \{2\}\}$, 3 and $\varnothing$: 3 elements. (b) 1 and $\{2\}$. (c) Both false.
7. One element for $x = 0$ and $x = 1$. Two elements for $x = -1$ only. Three elements for every other x.

WORKED SOLUTIONS

1. Let B be the set of real numbers x for which $3x - 2 \geq 13$. Decide whether each number is an element of B, and write each answer with $\in$ or $\notin$: 4, 7, 5, 0.

ANSWER

$4 \notin B, 7 \in B, 5 \in B, 0 \notin B$

SOLUTION

The rule gives a yes or no test: put the number in for x. The number is an element of B exactly when the result is at least 13.

- $x = 4$: $3 \cdot 4 - 2 = 10$, which is less than 13, so $4 \notin B$.
- $x = 7$: $3 \cdot 7 - 2 = 19$, which is at least 13, so $7 \in B$.
- $x = 5$: $3 \cdot 5 - 2 = 13$. That equals 13, and $\geq$ allows equality, so $5 \in B$.
- $x = 0$: $3 \cdot 0 - 2 = -2$, which is less than 13, so $0 \notin B$.

2. Decide whether each description gives a set. For each one that does not, say what question about membership has no clear answer.

(a) The whole numbers that are factors of 36.

(b) The big numbers less than 1000.

(c) The students in your class whose birthday is in March.

(d) The songs that are popular this year.

(e) The real numbers between 3 and 4.

ANSWER

(a), (c) and (e) give sets. (b) and (d) do not.

SOLUTION

A description gives a set exactly when every object gets a clear yes or no to "is it in?".

(a) Yes. For any whole number, "does it divide 36 with no remainder?" has a clear answer. (The set is $\{1, 2, 3, 4, 6, 9, 12, 18, 36\}$.)

(b) No. "Is 600 big?" has no settled answer, because "big" has no cutoff.

(c) Yes. Each student's birthday either is in March or is not.

(d) No. "Is this song popular?" has no cutoff, so some songs get no clear yes or no.

(e) Yes. Every real number either lies between 3 and 4 or does not. The set has infinitely many elements, and that is allowed: a set needs a clear test, not a short list.

3. Hannah rolls a six-sided die 5 times and writes down the set of numbers that came up: $\{2, 3, 6\}$.

(a) What is the largest number of times a 6 could have come up?

Free sample from math.wiki for classroom use. Do not post. math.wiki

(b) What is the smallest number of times a 6 could have come up?

(c) Carlos also rolls the die 5 times, and the set of numbers that came up is the same set. Must Carlos have rolled each number the same number of times as Hannah? Explain.

ANSWER
(a) 3 times. (b) 1 time. (c) No: the set shows only which numbers came up, not how many times each one did.

SOLUTION
The set lists exactly the numbers that came up, so each of 2, 3 and 6 came up at least once, and no other number came up at all.

(a) A 2 and a 3 take at least one roll each, which leaves at most 5 − 2 = 3 rolls for 6. That is possible: one 2, one 3 and 3 rolls of 6 give the set $\{2, 3, 6\}$. So the largest number is 3.

(b) 6 is an element of the set, so it came up at least once. Once is possible: one 6, one 3 and 3 rolls of 2. So the smallest number is 1.

(c) No. A set forgets how many times each object appears. For example, Hannah might have rolled 6 a total of 3 times while Carlos rolled 2 a total of 3 times. Both sets of results are $\{2, 3, 6\}$, but the counts are different.

4. Let G be the set of all numbers $3n^2 - 7$, where n is a whole number. Decide whether $68 \in G$ and whether $71 \in G$. Explain each answer.

ANSWER
$68 \in G$ and $71 \notin G$

SOLUTION
A number is in G exactly when some whole number n makes $3n^2 - 7$ equal to it.

For 68: $3n^2 - 7 = 68$ gives $3n^2 = 75$, so $n^2 = 25$. The whole number $n = 5$ works, so $68 \in G$.

For 71: $3n^2 - 7 = 71$ gives $3n^2 = 78$, so $n^2 = 26$. But $5^2 = 25$ and $6^2 = 36$, so 26 lies strictly between the squares of two whole numbers next to each other, and no whole number has square 26. So $71 \notin G$.

5. For which value of x does the set $\{6x - 1,\ -4x - 41\}$ have exactly one element? Write the set for that value of x.

ANSWER
$x = -4$, and the set is $\{-25\}$.

SOLUTION
Two listed objects make a set with one element exactly when they are the same object, because a repeat collapses. So the set has one element exactly when

$$6x - 1 = -4x - 41.$$

Collect the x terms on the left and the numbers on the right: $10x = -40$, so $x = -4$.

Check: $6 \cdot (-4) - 1 = -25$ and $-4 \cdot (-4) - 41 = -25$. The set is $\{-25, -25\} = \{-25\}$.

For any other x the two expressions are different numbers, so the set has two elements.

6. Let $W = \{\{1, \{2\}\}, 3, \varnothing\}$.

(a) List the elements of W and say how many there are.

(b) List every object that is an element of some element of W.

(c) True or false: $2 \in W$. True or false: 2 is an element of some element of W.

Free sample from math.wiki for classroom use. Do not post. math.wiki

ANSWER

(a) The elements are $\{1, \{2\}\}$, 3 and $\varnothing$: 3 elements. (b) 1 and $\{2\}$. (c) Both false.

SOLUTION

(a) The elements are the things written directly between the outer braces: the set $\{1, \{2\}\}$, the number 3 and the empty set $\varnothing$. So W has 3 elements.

(b) Look inside each element of W in turn. - $\{1, \{2\}\}$ has the elements 1 and $\{2\}$. - 3 is a number, a plain object, so it has no elements. - $\varnothing$ has no elements at all.

So the objects that are elements of some element of W are exactly 1 and $\{2\}$.

(c) $2 \in W$ is false: 2 is not one of the three elements from (a). The second statement is false too: 2 is not on the list in (b). The number 2 is an element of $\{2\}$, which is an element of $\{1, \{2\}\}$, which is an element of W, so 2 sits three layers down, and membership counts one layer only.

7. For which real numbers x does the set $\{x, x^2, x^3\}$ have exactly one element? For which does it have exactly two? Explain why every other real number x gives three elements.

ANSWER

One element for $x = 0$ and $x = 1$. Two elements for $x = -1$ only. Three elements for every other x.

SOLUTION

The number of elements is the number of different values among x, x^2 and x^3, so look for when two of them are equal.

At $x = 0$ the set is $\{0, 0, 0\} = \{0\}$: one element.

Now take $x \neq 0$, so both sides of an equation may be divided by x.

- $x = x^2$: dividing by x gives $1 = x$.
- $x^2 = x^3$: dividing by x twice gives $1 = x$.
- $x = x^3$: dividing by x gives $1 = x^2$, so $x = 1$ or $x = -1$.

At $x = 1$ the set is $\{1, 1, 1\} = \{1\}$: one element.

At $x = -1$ the set is $\{-1, 1, -1\} = \{-1, 1\}$: two elements. This is the only other value where any two of the expressions are equal, so it is the only value with two elements.

For every other x (not 0, 1 or –1), none of the three equations holds, so no two of x, x^2, x^3 are equal and the set has three different elements.

Free sample from math.wiki for classroom use. Do not post. math.wiki