Algebra 1 Unit 1 worksheet: Identity and inverse elements (answer key)

ANSWERS AT A GLANCE

1. (a) $-\frac{3}{4}$ (b) $\frac{3}{4}$ (c) $-\frac{25}{12}$
2. (a) The product is $\frac{259}{10}$, not 1, so the claim is wrong. (b) $\frac{8}{37}$
3. (a) The sum is $2x$, not 0, so Owen is wrong; the additive inverse is $-x+7$. (b) $4y-9$. (c) $\frac{w}{5}$, with $w \neq 0$. (d) $\frac{1}{3m}$, with $m \neq 0$.
4. (a) $-\frac{5}{8} \cdot \frac{5}{2} \cdot \left(-\frac{1}{5}\right) = \frac{5}{16}$ (b) $\frac{125}{16}$; not the same, so division is not associative.
5. (a) $x = -8$ (b) In part (a) the multiplication by $-\frac{4}{3}$ is the last thing done to x, so it is undone first; in $\frac{2}{3}x + 4 = 10$ the addition of 4 is done last.
6. (a) Not always: fails at $a = 0$. (b) Not always: fails at $a = 0$. (c) Always true. (d) Not always: fails at $a = 2$, for example. (e) Not always: fails at $a = -3$, for example. (f) Always true.
7. $a + (-1) \cdot a = 0$, so $(-1) \cdot a$ is an opposite of a; since a has only one opposite, $(-1) \cdot a = -a$.

WORKED SOLUTIONS

1. The reciprocal of a number is $-\frac{4}{3}$.

(a) What is the number?

(b) What is the additive inverse of the number?

(c) Find the sum of the number and its reciprocal.

ANSWER

(a) $-\frac{3}{4}$ (b) $\frac{3}{4}$ (c) $-\frac{25}{12}$

SOLUTION

(a) The reciprocal of a reciprocal is the original number, so the number is $-\frac{4}{3}$ turned over with its sign kept: $-\frac{3}{4}$. Check: $\left(-\frac{3}{4}\right)\left(-\frac{4}{3}\right) = 1$.

(b) The additive inverse is the opposite: $\frac{3}{4}$, since $-\frac{3}{4} + \frac{3}{4} = 0$.

(c) $-\frac{3}{4} + \left(-\frac{4}{3}\right) = -\frac{25}{12}$, using the common denominator $4 \cdot 3$.

2. Jack finds the reciprocal of $4\frac{5}{8}$ by turning only the fraction part over, and gets $4 + \frac{8}{5}$.

(a) Multiply $4\frac{5}{8}$ by $4 + \frac{8}{5}$ to test the claim.

(b) Find the correct reciprocal of $4\frac{5}{8}$.

ANSWER

(a) The product is $\frac{259}{10}$, not 1, so the claim is wrong. (b) $\frac{8}{37}$

SOLUTION

(a) Write both as fractions: $4\frac{5}{8} = \frac{37}{8}$ and $4 + \frac{8}{5} = \frac{28}{5}$. Their product is $\frac{37}{8} \cdot \frac{28}{5} = \frac{259}{10}$. A reciprocal must give the multiplicative identity 1, so this is not the reciprocal.

(b) The reciprocal has to undo the whole number: $\frac{37}{8}$ turned over is $\frac{8}{37}$. Check: $\frac{37}{8} \cdot \frac{8}{37} = 1$.

3. (a) Owen says the additive inverse of $x - 7$ is $x + 7$. Add the two expressions to test his claim, then give the correct additive inverse.

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(b) Find the additive inverse of $-4y + 9$, and check by adding.

(c) Find the multiplicative inverse of $\dfrac{5}{w}$, and say which value of w is not allowed.

(d) Find the multiplicative inverse of $3m$, and say which value of m is not allowed.

ANSWER

(a) The sum is $2x$, not 0, so Owen is wrong; the additive inverse is $-x + 7$. (b) $4y - 9$. (c) $\dfrac{w}{5}$, with $w \neq 0$. (d) $\dfrac{1}{3m}$, with $m \neq 0$.

SOLUTION

(a) $(x - 7) + (x + 7) = 2x$, which is 0 only when $x = 0$. The opposite must make 0 for every x: $(x - 7) + (-x + 7) = (x + (-x)) + (-7 + 7) = 0 + 0 = 0$. So the additive inverse is $-x + 7$: every term changes sign.

(b) Change the sign of every term: $4y - 9$. Check: $(-4y + 9) + (4y - 9) = 0 + 0 = 0$.

(c) Turn the fraction over: $\dfrac{w}{5}$. Check: $\dfrac{5}{w} \cdot \dfrac{w}{5} = \dfrac{5w}{5w} = 1$. The expression $\dfrac{5}{w}$ itself needs $w \neq 0$.

(d) $\dfrac{1}{3m}$, since $3m \cdot \dfrac{1}{3m} = 1$. When $m = 0$, $3m = 0$, and 0 has no reciprocal, so $m \neq 0$.

4. (a) Rewrite $-\dfrac{5}{8} \div \dfrac{2}{5} \div (-5)$, worked from left to right, as one product of three numbers, then evaluate it.

(b) Evaluate $-\dfrac{5}{8} \div \left(\dfrac{2}{5} \div (-5)\right)$. Is it the same number as in part (a)?

ANSWER

(a) $-\dfrac{5}{8} \cdot \dfrac{5}{2} \cdot \left(-\dfrac{1}{5}\right) = \dfrac{5}{16}$ (b) $\dfrac{125}{16}$; not the same, so division is not associative.

SOLUTION

(a) Dividing by a number is multiplying by its reciprocal. The reciprocal of $\dfrac{2}{5}$ is $\dfrac{5}{2}$ and the reciprocal of -5 is $-\dfrac{1}{5}$:

$$-\frac{5}{8} \cdot \frac{5}{2} \cdot \left(-\frac{1}{5}\right) = \frac{5}{16}.$$

(b) Inside the parentheses, $\dfrac{2}{5} \div (-5) = \dfrac{2}{5} \cdot \left(-\dfrac{1}{5}\right) = -\dfrac{2}{25}$. Its reciprocal is $-\dfrac{25}{2}$, so

$$-\frac{5}{8} \cdot \left(-\frac{25}{2}\right) = \frac{125}{16}.$$

The two answers differ: moving the parentheses changed the value, so division is not associative. Once a division is written as a multiplication, the factors can be regrouped freely.

5. (a) Solve $-\dfrac{4}{3}(x - 7) = 20$, naming the inverse you use at each step.

(b) In $\dfrac{2}{3}x + 4 = 10$ the opposite of 4 is used before the reciprocal of $\dfrac{2}{3}$. Explain why the order is the other way round in part (a).

ANSWER

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(a) $x = -8$ (b) In part (a) the multiplication by $-\frac{4}{3}$ is the last thing done to x, so it is undone first; in $\frac{2}{3}x + 4 = 10$ the addition of 4 is done last.

SOLUTION

(a) Here the last thing done to $x - 7$ is the multiplication, so undo it first. Multiply both sides by $-\frac{3}{4}$, the reciprocal of $-\frac{4}{3}$. Their product is 1 (inverse property of multiplication), and 1 times a number leaves it unchanged (identity property of multiplication):

$$x - 7 = -\frac{3}{4} \cdot 20 = -15.$$

Add 7, the opposite of −7, to both sides. The two make 0 (inverse property of addition), and adding 0 changes nothing (identity property of addition):

$$x = -15 + 7 = -8.$$

Check: $-\frac{4}{3}(-8 - 7) = -\frac{4}{3} \cdot (-15) = 20$.

(b) Inverses undo the steps in reverse order. In part (a), −7 is added to x first and the sum is then multiplied by $-\frac{4}{3}$, so the multiplication is undone first. In $\frac{2}{3}x + 4 = 10$, x is multiplied first and 4 is added last, so the opposite of 4 comes first.

6. Decide whether each statement is true for every real number a it mentions. If it is not, give a value of a that makes it fail.

(a) $0 \div a = 0$

(b) $a \div a = 1$

(c) $-(-a) = a$

(d) $1 \div a = a$

(e) $-a$ is a negative number.

(f) $\frac{1}{1/a} = a$ for every $a \neq 0$

ANSWER

(a) Not always: fails at $a = 0$. (b) Not always: fails at $a = 0$. (c) Always true. (d) Not always: fails at $a = 2$, for example. (e) Not always: fails at $a = -3$, for example. (f) Always true.

SOLUTION

(a) For $a = 0$, $0 \div 0$ would mean $0 \cdot \frac{1}{0}$, and 0 has no reciprocal, so it has no value. For every other a it is true.

(b) Fails at $a = 0$ for the same reason. For $a \neq 0$, $a \div a = a \cdot \frac{1}{a} = 1$ by the inverse property.

(c) True: a adds to $-a$ to give 0, and $-a$ has only one opposite, so $-(-a) = a$.

(d) Fails at $a = 2$: $1 \div 2 = \frac{1}{2} \neq 2$. (It holds only for $a = 1$ and $a = -1$.) This is why 1 is not an identity for division.

(e) Fails at $a = -3$: $-a = 3$, which is positive. (It also fails at $a = 0$, since −0 = 0.)

(f) True: $\frac{1}{1/a}$ is the reciprocal of $\frac{1}{a}$, and $\frac{1}{a}$ has only one reciprocal, which is a.

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7. Explain why $(-1)\cdot a = -a$ for every real number a. Start from $a + (-1)\cdot a$ and give the property behind each step. You may use the fact, proved in the lesson, that $0 \cdot b = 0$ for every b.

ANSWER

$a + (-1)\cdot a = 0$, so $(-1)\cdot a$ is an opposite of a; since a has only one opposite, $(-1)\cdot a = -a$.

SOLUTION

$$\begin{aligned} a + (-1)\cdot a &= 1\cdot a + (-1)\cdot a && \text{identity property of multiplication} \\ &= \big(1 + (-1)\big)\cdot a && \text{distributive property} \\ &= 0 \cdot a && \text{inverse property of addition} \\ &= 0 && 0 \cdot b = 0 \text{ for every } b \end{aligned}$$

So $(-1)\cdot a$ is a number that, added to a, gives 0: it is an opposite of a. Every number has only one opposite, and that opposite is $-a$. Therefore $(-1)\cdot a = -a$.

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