Algebra 1 Unit 1 worksheet: Interpreting the parts of an expression (answer key)

ANSWERS AT A GLANCE

1. (a) Last step divide: a quotient, numerator $x + 3$, denominator $2x$. (b) Last step raise to a power: a power, base $2x - 1$, exponent 3. (c) Last step subtract: a sum of the terms x^2 and $-4(x + 1)$; the second term has factors -4 and $x + 1$. (d) Last step multiply: a product of the factors 6, x^2 and $x - 7$.
2. (a) 1900 liters is the water in the tank at the start; 25 is the liters pumped out each minute; $25t$ is the liters pumped out in t minutes. (b) 1025 liters (c) 76 minutes (d) $0 \le t \le 76$
3. (a) The total of the first 5 scores; a mean is a total divided by a count, so the old scores enter as their total, 370. (b) The numerator is the total of all 6 scores; the denominator is the number of tests. (c) $x = 92$
4. (a) The numerator is the whole bill in dollars, the denominator is the number of guests sharing it, and $10n$ is the cost of food for n guests. (b) 25 dollars (c) The numerator is $10n$ plus a positive 300, so it is more than $10n$; dividing by n gives more than 10.
5. (a) A product: p is the price of one ticket in dollars and $380 - 8p$ is the number of tickets sold. (b) 380 is the number of tickets that would go if they were free; 8 is how many fewer tickets sell for each dollar added to the price. (c) 300 tickets, 3000 dollars
6. (a) The square is subtracted, so taking away a number that is at least 0 makes the result at most 4: 4 is the largest value, at $x = 3$, not the smallest. (b) For example $x = 14$, which gives -117 (any $x \ge 14$ or $x \le -8$ works). (c) No: the square can be made as large as we like, so the expression can be made as low as we like.
7. (a) Writing u for $x + 2$, the left side is $u(x + 5) - u(x + 3) = u\big((x + 5) - (x + 3)\big) = u \cdot 2$, which is $2(x + 2)$. (b) 200
8. (a) Never positive; 0 only at $x = -1$. (b) Always positive. (c) Always negative. (d) Never negative; 0 only at $x = 0$. (e) Both signs: it is -1 at $x = 2$ and 3 at $x = 0$; it is 0 at $x = 1$ and $x = 3$.

WORKED SOLUTIONS

1. For each expression, say what the last step is when it is worked out in the order of operations, and so whether it is a sum, a product, a quotient or a power. Then name its top-level parts as asked.

   (a) $\dfrac{x + 3}{2x}$: name the numerator and the denominator.

   (b) $(2x - 1)^3$: name the base and the exponent.

   (c) $x^2 - 4(x + 1)$: name the terms, and the factors of the second term.

   (d) $6x^2(x - 7)$: name the factors.

   ANSWER

   (a) Last step divide: a quotient, numerator $x + 3$, denominator $2x$. (b) Last step raise to a power: a power, base $2x - 1$, exponent 3. (c) Last step subtract: a sum of the terms x^2 and $-4(x + 1)$; the second term has factors -4 and $x + 1$. (d) Last step multiply: a product of the factors 6, x^2 and $x - 7$.

   SOLUTION

   (a) The fraction bar groups the top and the bottom, so both are worked out first and the division comes last. The expression is a quotient: the numerator $x + 3$ divided by the denominator $2x$.

   (b) The part in parentheses is worked out first, then cubed. The last step is raising to the power 3, so the expression is a power with base $2x - 1$ and exponent 3: three factors of $2x - 1$ multiplied together.

   (c) The square and the product $4(x + 1)$ are worked out before the subtraction, so the last step is subtracting. The terms, with their signs, are x^2 and $-4(x + 1)$. The second term is a product of -4 and the sum in parentheses, $x + 1$.

   (d) The power x^2 and the part in parentheses are worked out first, then everything is multiplied. It is a product with factors 6, x^2 and $x - 7$.

2. A pump empties a water tank. After t minutes of pumping, the water left in the tank is $1900 - 25t$ liters.

   (a) Say what 1900, 25 and $25t$ each stand for, with their units.

Free sample from math.wiki for classroom use. Do not post. math.wiki

(b) How much water is left after 35 minutes?

(c) After how many minutes is the tank empty?

(d) For which values of t does the expression describe the tank?

ANSWER

(a) 1900 liters is the water in the tank at the start; 25 is the liters pumped out each minute; $25t$ is the liters pumped out in t minutes. (b) 1025 liters (c) 76 minutes (d) $0 \leq t \leq 76$

SOLUTION

(a) The expression is a difference: the starting amount minus the amount pumped out. 1900 is the water at the start, in liters. The coefficient 25 is the rate, in liters per minute, and t is the time in minutes, so the term $25t$ is the water pumped out so far, in liters.

(b) After 35 minutes the pump has taken out 25 × 35 = 875 liters, so 1900 − 875 = 1025 liters are left.

(c) The tank is empty when all 1900 liters are pumped out: $25t = 1900$, so $t = 1900 \div 25 = 76$ minutes.

(d) From $t = 0$, when pumping starts, to $t = 76$, when the tank is empty. After that the expression is negative and describes no tank.

3. After 5 tests, Hannah's mean score is 74. If Hannah scores x on the next test, the new mean will be

$$\frac{5 \cdot 74 + x}{6}$$

(a) What does $5 \cdot 74$ stand for? Why is it in the numerator rather than just 74?

(b) What do the whole numerator and the denominator 6 stand for?

(c) What score on the next test makes the new mean exactly 77?

ANSWER

(a) The total of the first 5 scores; a mean is a total divided by a count, so the old scores enter as their total, 370. (b) The numerator is the total of all 6 scores; the denominator is the number of tests. (c) $x = 92$

SOLUTION

(a) A mean of 74 over 5 tests means the scores add to $5 \cdot 74 = 370$. The new mean needs the total of all the scores, so the first 5 tests enter as their total, not as their mean.

(b) The numerator $5 \cdot 74 + x$ is the old total plus the new score: the total of all 6 scores. The denominator 6 is the number of tests, so the quotient is the new mean.

(c) For a mean of 77 over 6 tests the total must be 6 × 77 = 462. So $370 + x = 462$ and $x = 462 - 370 = 92$.

4. Hannah books a party room. The room costs 300 dollars to rent, and food costs 10 dollars a guest. With n guests sharing the whole bill equally, each guest pays

$$\frac{300 + 10n}{n}$$

dollars.

(a) What do the numerator, the denominator and the term $10n$ stand for?

(b) How much does each guest pay when 20 guests come?

(c) Explain from the parts of the expression why each guest always pays more than 10 dollars, however many come.

ANSWER

Free sample from math.wiki for classroom use. Do not post. math.wiki

(a) The numerator is the whole bill in dollars, the denominator is the number of guests sharing it, and $10n$ is the cost of food for n guests. (b) 25 dollars (c) The numerator is $10n$ plus a positive 300, so it is more than $10n$; dividing by n gives more than 10.

SOLUTION

(a) The last step is dividing, so this is a quotient. The numerator $300 + 10n$ is the whole bill: the room rental plus the food. The term $10n$ is the food, 10 dollars for each of n guests. The denominator n is the number of guests the bill is split between.

(b) With 20 guests the bill is 300 + 10 × 20 = 300 + 200 = 500 dollars, and each guest pays 500 ÷ 20 = 25 dollars.

(c) If the bill were only food, it would be $10n$, and each share would be $10n \div n = 10$ dollars. The real numerator is $10n$ plus 300, more than the food alone, so each share is more than 10 dollars. The extra is each guest's part of the room, 300 divided by n (here 15 dollars), which shrinks as more guests come but never reaches 0.

5. A theater finds that when a ticket costs p dollars, it sells $380 - 8p$ tickets for a show. So the money it takes from ticket sales is

$$p(380 - 8p)$$

dollars.

(a) Is this a sum or a product at the top level? Say what each top-level part stands for.

(b) In the parentheses, what do 380 and 8 mean?

(c) How many tickets are sold, and how much money is taken, when tickets cost 10 dollars?

ANSWER

(a) A product: p is the price of one ticket in dollars and $380 - 8p$ is the number of tickets sold. (b) 380 is the number of tickets that would go if they were free; 8 is how many fewer tickets sell for each dollar added to the price. (c) 300 tickets, 3000 dollars

SOLUTION

(a) The part in parentheses is worked out first and then multiplied by p, so the expression is a product of two factors: the price per ticket, p dollars, and the number of tickets sold, $380 - 8p$. Price times number sold is the money taken.

(b) At $p = 0$ the part in parentheses is 380, so 380 is how many tickets would go at no price. Each dollar on the price takes away 8 more from the count, so 8 is the number of sales lost per dollar.

(c) At $p = 10$: tickets sold 380 − 8 × 10 = 380 − 80 = 300, and money taken 10 × 300 = 3000 dollars.

6. Owen looks at $4 - (x - 3)^2$ and writes: "A square is never negative, so the smallest value of this expression is 4."

(a) What is wrong with Owen's reasoning? What does the fact about squares really tell you about this expression?

(b) Find a value of x that makes the expression less than −100.

(c) Does the expression have a smallest value? Explain.

ANSWER

(a) The square is subtracted, so taking away a number that is at least 0 makes the result at most 4: 4 is the largest value, at $x = 3$, not the smallest. (b) For example $x = 14$, which gives −117 (any $x \geq 14$ or $x \leq -8$ works). (c) No: the square can be made as large as we like, so the expression can be made as low as we like.

Free sample from math.wiki for classroom use. Do not post. math.wiki

SOLUTION

(a) The expression is 4 minus a square. The square is never negative, so the most it can take away is nothing. That makes 4 the **largest** value, reached when $x - 3 = 0$, at $x = 3$. Owen read "the square is at least 0" as if the square were added.

(b) We need the square to be more than 104. Take $x - 3 = 11$, so $x = 14$: then $4 - 11^2 = 4 - 121 = -117$, which is less than −100.

(c) No. As x moves away from 3, the square $(x - 3)^2$ grows without end, and it is being subtracted, so the expression falls below any number we name. There is no smallest value.

7. (a) Without multiplying anything out, explain why

$$(x + 2)(x + 5) - (x + 2)(x + 3) = 2(x + 2)$$

for every real number x. (Hint: read the sum in parentheses, $x + 2$, as one number.)

(b) Use part (a) to find the value of $(x + 2)(x + 5) - (x + 2)(x + 3)$ when $x = 98$, with no long multiplication.

ANSWER

(a) Writing u for $x + 2$, the left side is $u(x + 5) - u(x + 3) = u\big((x + 5) - (x + 3)\big) = u \cdot 2$, which is $2(x + 2)$. (b) 200

SOLUTION

(a) The left side is a difference of two terms, and each term is a product with the factor $x + 2$. Read that sum as one number u. Then the left side is $u(x + 5) - u(x + 3)$, and by the distributive property, right to left, this is $u\big((x + 5) - (x + 3)\big)$. The expression in the big parentheses is $x + 5 - x - 3 = 2$, so the left side is $u \cdot 2 = 2(x + 2)$.

In words: $x + 5$ copies of the number u, take away $x + 3$ copies of it, leaves 2 copies.

(b) At $x = 98$ the sum $x + 2$ is 100, so the value is 2 × 100 = 200.

8. Without trying values, use the structure of each expression to decide whether it is always positive, always negative, never positive, never negative, or can take both signs. Where it can equal 0, say for which x.

(a) $-(x + 1)^2$

(b) $(x - 3)^2 + 1$

(c) $\dfrac{-5}{x^2 + 1}$

(d) $x^2(x^2 + 4)$

(e) $(x - 2)^2 - 1$

ANSWER

(a) Never positive; 0 only at $x = -1$. (b) Always positive. (c) Always negative. (d) Never negative; 0 only at $x = 0$. (e) Both signs: it is −1 at $x = 2$ and 3 at $x = 0$; it is 0 at $x = 1$ and $x = 3$.

SOLUTION

(a) This is the opposite of a square. The square $(x + 1)^2$ is never negative, so its opposite is never positive. It is 0 only when $x + 1 = 0$, at $x = -1$.

(b) A square, which is at least 0, plus 1: at least 1, so always positive.

(c) The denominator is $x^2 + 1$, at least 1, so positive. A negative number divided by a positive one is negative, so the quotient is always negative.

(d) A product of x^2, never negative, and $x^2 + 4$, at least 4 and so positive. The product is never negative, and it is 0 only when the factor x^2 is 0, at $x = 0$.

Free sample from math.wiki for classroom use. Do not post. math.wiki

(e) A square minus 1. The square can be 0, which makes the whole -1, at $x = 2$; and it can be large, for example $(0 - 2)^2 - 1 = 3$. So it takes both signs. It is 0 when the square is 1, so $x - 2 = 1$ or $x - 2 = -1$: at $x = 3$ and $x = 1$.

Free sample from math.wiki for classroom use. Do not post. math.wiki