Algebra 1 Unit 1 worksheet: Units, quantities and a sensible level of accuracy (answer key)

ANSWERS AT A GLANCE

1. (a) 6800 m (b) The three lengths are not in the same unit, so their numbers cannot be added as they stand; the 6 is kilometers, not meters.
2. (a) 60 m^2, an area (b) 90 km/h, a speed (c) 5 min, a time (d) 180 km, a length (e) cannot be worked out: a mass and a length cannot be added.
3. (a) $\frac{\mathrm{s}^2}{\mathrm{h}}$ (b) $5400\ \mathrm{s} \times \frac{1\ \mathrm{h}}{3600\ \mathrm{s}} = 1.5\ \mathrm{h}$
4. 1 h 33 min
5. (a) about 1.4 km (b) Only 7.3: the 10 and the 1850 are counts, so they are exact.
6. (a) Days, hours and minutes each appear once on top and once underneath, leaving seconds. (b) An hour has 60 minutes, not 100; the month has 2,592,000 s. (c) Units catch a factor written the wrong way up or a missing conversion; they cannot catch a wrong number inside a factor whose units are right.
7. 144 m^3
8. About 13 dollars a year. The 5 loads, the 52 weeks and the 1000 liters in a cubic meter are exact (the price is exact too); the 12 liters limits the answer to two figures.

WORKED SOLUTIONS

1. Hannah's hike has three parts: 6 km along a road, 250 m up a trail and 550 m across a ridge.

(a) Find the total length of the hike in meters.

(b) A friend adds 6 + 250 + 550 and says the hike is 806 m long. Explain what went wrong.

ANSWER

(a) 6800 m (b) The three lengths are not in the same unit, so their numbers cannot be added as they stand; the 6 is kilometers, not meters.

SOLUTION

(a) Only quantities with the same unit can be added, so first write the road in meters:

$$6\ \mathrm{km} \times \frac{1000\ \mathrm{m}}{1\ \mathrm{km}} = 6000\ \mathrm{m}$$

Then 6000 m + 250 m + 550 m = 6800 m.

(b) The friend added kilometers to meters. The 6 counts kilometers, and each kilometer is 1000 meters, so the sum 806 is not a number of anything. Every term needs the same unit before adding.

2. Work out each result with its unit, and say what kind of quantity it is (a length, an area, a time, a speed, and so on). If a result cannot be worked out, say why.

(a) $12\ \mathrm{m} \times 5\ \mathrm{m}$

(b) $360\ \mathrm{km} \div 4\ \mathrm{h}$

(c) $45\ \mathrm{L} \div \frac{9\ \mathrm{L}}{1\ \mathrm{min}}$

(d) $2\ \mathrm{h} \times \frac{90\ \mathrm{km}}{1\ \mathrm{h}}$

(e) $4\ \mathrm{kg} + 0.5\ \mathrm{m}$

ANSWER

(a) 60 m^2, an area (b) 90 km/h, a speed (c) 5 min, a time (d) 180 km, a length (e) cannot be worked out: a mass and a length cannot be added.

SOLUTION

Units multiply, divide and cancel like letters.

(a) $12 \times 5 = 60$ and $\mathrm{m} \times \mathrm{m} = \mathrm{m}^2$, so 60 m^2, an area.

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(b) $360 \div 4 = 90$ with unit $\dfrac{\text{km}}{\text{h}}$: 90 km/h, a speed.

(c) Dividing by a fraction is multiplying by its reciprocal: $45\ \text{L} \times \dfrac{1\ \text{min}}{9\ \text{L}} = 5\ \text{min}$. Liters cancel, leaving a time.

(d) $2\ \text{h} \times \dfrac{90\ \text{km}}{1\ \text{h}} = 180\ \text{km}$. Hours cancel, leaving a length.

(e) Only quantities with the same unit can be added, and a mass and a length are different kinds of quantity, so no unit change can make them match. The sum has no meaning.

3. Samir wants to write the length of a 5400-second video in hours. He writes

$$5400\ \text{s} \times \frac{3600\ \text{s}}{1\ \text{h}} = 19{,}440{,}000$$

(a) What unit does his product actually have?

(b) Write the correct chain and give the length of the video in hours.

ANSWER

(a) $\dfrac{\text{s}^2}{\text{h}}$ (b) $5400\ \text{s} \times \dfrac{1\ \text{h}}{3600\ \text{s}} = 1.5\ \text{h}$

SOLUTION

(a) Nothing cancels: seconds times seconds is s^2, and hours stay underneath, so the unit is $\dfrac{\text{s}^2}{\text{h}}$. That measures nothing anybody wants, which is the sign the factor is upside down.

(b) Seconds must cancel, so seconds go underneath in the factor:

$$5400\ \text{s} \times \frac{1\ \text{h}}{3600\ \text{s}} = \frac{5400}{3600}\ \text{h} = 1.5\ \text{h}$$

The video is 1.5 hours long, that is, 1 hour 30 minutes.

4. Hannah runs a machine that labels 30 bottles every 3 minutes. How long does it take to label 930 bottles? Give the time in hours and minutes.

ANSWER

1 h 33 min

SOLUTION

The rate is $\dfrac{30\ \text{bottles}}{3\ \text{min}}$. Bottles must cancel to leave a time, so the rate is used the other way up, with bottles underneath:

$$930\ \text{bottles} \times \frac{3\ \text{min}}{30\ \text{bottles}} = \frac{930 \times 3}{30}\ \text{min} = 93\ \text{min}$$

In hours: $93 \div 60$ is 1 with remainder 33, so the job takes 1 hours 33 minutes.

5. Maya measures her walking stride: 10 strides cover 7.3 m, measured with a tape to the nearest tenth of a meter. She counts 1850 strides on her walk to school.

(a) How long is her walk in kilometers? Give the answer to a sensible accuracy.

(b) Which of the numbers 10, 7.3 and 1850 limit the accuracy of the answer, and why?

ANSWER

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(a) about 1.4 km (b) Only 7.3: the 10 and the 1850 are counts, so they are exact.

SOLUTION

(a) One stride is $\dfrac{7.3\ \text{m}}{10\ \text{strides}}$. Strides cancel, then meters:

$$1850\ \text{strides} \times \frac{7.3\ \text{m}}{10\ \text{strides}} \times \frac{1\ \text{km}}{1000\ \text{m}} = 1.3505\ \text{km}$$

(b) Counts are exact numbers, so 10 and 1850 never limit the answer. The 7.3 is measured, with two significant figures, so the answer keeps two: about 1.4 km. (The model also assumes every stride is the same length, which is another reason not to claim more.)

6. Hiro wants the number of seconds in a 30-day month. He writes

$$30\ \text{days} \times \frac{24\ \text{h}}{1\ \text{day}} \times \frac{100\ \text{min}}{1\ \text{h}} \times \frac{60\ \text{s}}{1\ \text{min}} = 4{,}320{,}000\ \text{s}$$

(a) Show that the units in his chain cancel to leave seconds.

(b) His answer is still wrong. Find the mistake and the correct number of seconds.

(c) What can checking the units catch in a set-up like this, and what can it not catch?

ANSWER

(a) Days, hours and minutes each appear once on top and once underneath, leaving seconds. (b) An hour has 60 minutes, not 100; the month has 2,592,000 s. (c) Units catch a factor written the wrong way up or a missing conversion; they cannot catch a wrong number inside a factor whose units are right.

SOLUTION

(a) Days cancel between the first quantity and the first factor, hours between the first and second factors, minutes between the second and third. Only seconds remain.

(b) The factor $\dfrac{100\ \text{min}}{1\ \text{h}}$ has the right units but is not equal to one: 1 h = 60 min. With the true factor,

$$30 \times 24 \times 60 \times 60 = 2{,}592{,}000\ \text{s}$$

(c) A unit check shows a fraction written upside down (the leftover unit is wrong) or a conversion left out (two units fail to cancel). It cannot see a wrong value in a factor with the right units, as here, so every conversion factor must also be a true equality.

7. A pump moves 100 liters of water per minute. Running nonstop at that rate, how many cubic meters of water does it move in one day? Write the whole conversion as one chain of factors before doing any arithmetic.

ANSWER

$144\ \text{m}^3$

SOLUTION

Minutes must cancel on the way to days, then liters on the way to cubic meters ($1\ \text{m}^3 = 1000\ \text{L}$):

$$\frac{100\ \text{L}}{1\ \text{min}} \times \frac{60\ \text{min}}{1\ \text{h}} \times \frac{24\ \text{h}}{1\ \text{day}} \times \frac{1\ \text{m}^3}{1000\ \text{L}}$$

Minutes, hours and liters cancel, leaving cubic meters per day. The arithmetic: 100 × 60 = 6000, then 6000 × 24 = 144000, then 144000 ÷ 1000 = 144.

The pump moves $144\ \text{m}^3$ of water a day.

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8. Mason's dishwasher uses 12 liters of water per load, a figure from its label given to two significant figures. The family runs 5 loads a week, and water costs 4.20 dollars per cubic meter. Taking a year as 52 weeks, about how much does the dishwasher's water cost in a year? Say which numbers are exact and give the cost to a sensible accuracy.

ANSWER

About 13 dollars a year. The 5 loads, the 52 weeks and the 1000 liters in a cubic meter are exact (the price is exact too); the 12 liters limits the answer to two figures.

SOLUTION

Loads cancel against loads, weeks against weeks, liters against liters, and cubic meters against cubic meters:

$$\frac{12\ \text{L}}{1\ \text{load}} \times \frac{5\ \text{loads}}{1\ \text{week}} \times \frac{52\ \text{weeks}}{1\ \text{year}} \times \frac{1\ \text{m}^3}{1000\ \text{L}} \times \frac{4.20\ \text{dollars}}{1\ \text{m}^3}$$

Step by step: $12 \times 5 \times 52 = 3120$ liters a year, which is $3.12\ \text{m}^3$, and $3.12 \times 4.20 = 13.104$ dollars a year.

The counts (5 loads, 52 weeks) and the definition $1\ \text{m}^3 = 1000\ \text{L}$ are exact, and a price is exact as charged. The only measured input is the 12 liters, with two significant figures, so the answer keeps two: about 13 dollars a year.

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