Algebra 1 Unit 1 worksheet: Writing sets: roster, rule and the empty set (answer key)

ANSWERS AT A GLANCE

1. $\{12, 16, 20, \ldots, 48\}$, with 10 elements
2. (a) $\{-3, -2, -1, 0, 1, 2, 3\}$, 7 elements. (b) $\varnothing$, 0 elements. (c) $\{1, 2, 3, 6\}$, 4 elements. (d) $\{0, 1\}$, 2 elements.
3. (a) $B = \{72, 79, 86, \ldots, 163\}$. (b) Yes, $93 \in B$. (c) No, $191 \notin B$.
4. (a) The integers that are 2 more than a multiple of 4. (b) Infinite. (c) $42 \in S$, $-15 \notin S$, $-22 \in S$.
5. Three groups: (a), (b) and (e) name $\{0\}$; (c), (f) and (h) name $\varnothing$; (d) and (g) name $\{\varnothing\}$.
6. (b), (d) and (e) name the set. (a) does not: it lets in $1 = 2^0$. (c) does not: it lets in 6 (and every other even number that is not a power of 2).
7. (a) For example 0.25. (b) 9 elements. (c) Between any two numbers there is another, so the set has no step from one element to the next for the dots to repeat; it is described by a rule in words: the numbers greater than 0 and less than 1.
8. (a) $\{0, 1, 2\}$, finite. (b) $\{21, 22, 23, \ldots\}$, infinite. (c) $\{\ldots, 0, 1, 2\}$, infinite. (d) $\varnothing$, finite.

WORKED SOLUTIONS

1. Write the set of multiples of 4 that are greater than 11 and less than 50 as a roster with an ellipsis. How many elements does it have?

ANSWER

$\{12, 16, 20, \ldots, 48\}$, with 10 elements

SOLUTION

The first multiple of 4 above 11 is $12 = 4 \cdot 3$, and the last one below 50 is $48 = 4 \cdot 12$. The entries go up by 4:

$$\{12, 16, 20, \ldots, 48\}$$

The elements are 4 times each whole number from 3 to 12, so there are $12 - 3 + 1 = 10$ of them.

2. Write each set as a roster and say how many elements it has.

(a) The integers whose square is less than 10.

(b) The whole-number solutions of $2x + 3 = 10$.

(c) The whole numbers that are factors of both 12 and 18.

(d) The integers n whose square is equal to n itself.

ANSWER

(a) $\{-3, -2, -1, 0, 1, 2, 3\}$, 7 elements. (b) $\varnothing$, 0 elements. (c) $\{1, 2, 3, 6\}$, 4 elements. (d) $\{0, 1\}$, 2 elements.

SOLUTION

(a) The squares 0, 1, 4, 9 are less than 10, but $4^2 = 16$ is not, and a negative integer has the same square as its opposite. So the set is $\{-3, -2, -1, 0, 1, 2, 3\}$, with 7 elements. Do not forget 0: its square, 0, is less than 10.

(b) Subtracting 3 gives $2x = 7$, so $x = 3.5$, which is not a whole number. No whole number is a solution, so the set is $\varnothing$, with 0 elements.

(c) The factors of 12 are 1, 2, 3, 4, 6, 12 and the factors of 18 are 1, 2, 3, 6, 9, 18. Those in both lists give $\{1, 2, 3, 6\}$, with 4 elements.

(d) $0^2 = 0$ and $1^2 = 1$, so 0 and 1 are in the set. A negative integer has a positive square, so its square is not itself. An integer n of 2 or more has $n^2 = n \cdot n \geq 2n$, which is more than n. The set is $\{0, 1\}$, with 2 elements.

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3. Hannah puts 65 dollars in a piggy bank and adds 7 dollars at the end of every week, taking nothing out. Let B be the set of the amounts in the piggy bank, in dollars, at the end of weeks 1 to 14.

(a) Write B as a roster with an ellipsis.

(b) Is 93 an element of B?

(c) Is 191 an element of B?

ANSWER

(a) $B = \{72, 79, 86, \ldots, 163\}$. (b) Yes, $93 \in B$. (c) No, $191 \notin B$.

SOLUTION

(a) After week 1 the amount is 65 + 7 = 72, and each week adds 7. After week 14 it is $65 + 14 \cdot 7 = 163$, and the list stops there:

$$B = \{72, 79, 86, \ldots, 163\}$$

(b) $93 - 65 = 4 \cdot 7$, so this is the amount after week 4, which is one of weeks 1 to 14. Yes, $93 \in B$.

(c) $191 - 65 = 18 \cdot 7$, so this would be the amount after week 18. The set stops at week 14, so no, $191 \notin B$, even though it fits the pattern.

4. Let $S = \{\ldots, -6, -2, 2, 6, \ldots\}$, a list with dots at both ends.

(a) Describe S by a rule in words.

(b) Is S finite or infinite?

(c) Decide whether each of 42, −15 and −22 is an element of S.

ANSWER

(a) The integers that are 2 more than a multiple of 4. (b) Infinite. (c) $42 \in S$, $-15 \notin S$, $-22 \in S$.

SOLUTION

(a) Each entry is 4 more than the one before, and the dots at both ends say the pattern carries on forever in both directions. Since $2 = 4 \cdot 0 + 2$, every entry is 2 plus a multiple of 4, the multiple being negative, zero or positive. So S is the set of integers that are 2 more than a multiple of 4.

(b) Infinite: adding 4 to any element gives another element.

(c) Subtract 2 and ask whether a multiple of 4 is left.

- $42 - 2 = 40 = 4 \cdot 10$, so $42 \in S$.
- $-15 - 2 = -17 = 4 \cdot (-4) - 1$, which is 1 less than a multiple of 4 and so not a multiple. So $-15 \notin S$.
- $-22 - 2 = -24 = 4 \cdot (-6)$, so $-22 \in S$.

5. Sort these eight descriptions into groups, so that two descriptions are in the same group exactly when they name the same set. Write the set each group names.

(a) The whole numbers n for which $n + 5 = 5$.

(b) The integers strictly between −1 and 1.

(c) The prime numbers that are multiples of 6.

(d) $\{\varnothing\}$

(e) The solutions of $2x = x$.

(f) The integers whose square is −4.

(g) {{ }}

(h) { }

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ANSWER
Three groups: (a), (b) and (e) name $\{0\}$; (c), (f) and (h) name $\varnothing$; (d) and (g) name $\{\varnothing\}$.

SOLUTION
Work out each set first.

- (a) $n + 5 = 5$ gives $n = 0$, a whole number: $\{0\}$.
- (b) The only integer strictly between -1 and 1 is 0: $\{0\}$.
- (c) A multiple of 6 such as 6, 12 or 18 has 2 and 3 as factors, so none is prime: $\varnothing$.
- (d) One element, the empty set: $\{\varnothing\}$.
- (e) Subtracting x from both sides gives $x = 0$: $\{0\}$.
- (f) The square of any integer is 0 or positive, never -4: $\varnothing$.
- (g) $\{\ \}$ is another way to write $\varnothing$, so this is a set whose one element is the empty set: $\{\varnothing\}$.
- (h) Braces with nothing between them: $\varnothing$.

So there are three groups: (a), (b) and (e) name $\{0\}$; (c), (f) and (h) name $\varnothing$; (d) and (g) name $\{\varnothing\}$. These three sets are different: $\varnothing$ has no elements, while $\{0\}$ and $\{\varnothing\}$ each have one, and their elements differ.

6. Five students each propose a rule in words for the set $\{2, 4, 8, 16, 32\}$. For each rule, decide whether it names exactly this set. If it does not, name one object it gets wrong.

(a) The powers of 2 less than 40.

(b) The even factors of 32.

(c) The even whole numbers from 2 to 32.

(d) The numbers you get by starting at 2 and doubling, for as long as the result is at most 32.

(e) The factors of 64 that are greater than 1 and less than 64.

ANSWER
(b), (d) and (e) name the set. (a) does not: it lets in $1 = 2^0$. (c) does not: it lets in 6 (and every other even number that is not a power of 2).

SOLUTION
A rule names the set only if it lets in every element and nothing else, so test each one both ways.

(a) 2, 4, 8, 16, 32 are powers of 2 less than 40, but so is $1 = 2^0$, and 1 is not in the set. Not the set.

(b) The factors of 32 are 1, 2, 4, 8, 16, 32. The even ones are 2, 4, 8, 16, 32, exactly the set.

(c) This lets in 6, which is even and between 2 and 32 but not in the set. Not the set.

(d) Starting at 2 and doubling gives 2, 4, 8, 16, 32; the next, 64, is more than 32. Exactly the set.

(e) The factors of 64 are 1, 2, 4, 8, 16, 32, 64. Leaving out 1 and 64 gives 2, 4, 8, 16, 32, exactly the set.

7. Liam wants to name the set of all numbers greater than 0 and less than 1, and writes $\{0.1, 0.2, 0.3, \ldots, 0.9\}$.

(a) Name a number greater than 0 and less than 1 that is not in Liam's set.

(b) How many elements does Liam's set have?

(c) Explain why no roster with an ellipsis can name the set Liam meant, and describe that set correctly.

ANSWER
(a) For example 0.25. (b) 9 elements. (c) Between any two numbers there is another, so the set has no step from one element to the next for the dots to repeat; it is described by a rule in words: the numbers greater than 0 and less than 1.

SOLUTION

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(a) The entries go up by 0.1, so 0.25, which lies between 0.2 and 0.3, is greater than 0 and less than 1 but is not an entry. (Any number such as 0.05 or $\frac{1}{3}$ also works.)

(b) The entries are $0.1 \cdot 1$, $0.1 \cdot 2$, ..., $0.1 \cdot 9$, so Liam's set is finite with 9 elements.

(c) An ellipsis repeats a step from one entry to the next. Whatever first two entries are written, the number halfway between them is in the set Liam meant but is skipped by the step, so some element is always left out. The set has no "next" member at all. It has to be described by a rule: the numbers greater than 0 and less than 1. It is infinite.

8. Let $A = \{3, 8, 20\}$. Write each set as a complete roster or as a roster with an ellipsis, and say whether it is finite or infinite.

(a) The whole numbers that are less than every element of A.

(b) The whole numbers that are greater than every element of A.

(c) The integers that are less than every element of A.

(d) The whole numbers that are greater than every element of A and also less than every element of A.

ANSWER

(a) $\{0, 1, 2\}$, finite. (b) $\{21, 22, 23, \dots\}$, infinite. (c) $\{\dots, 0, 1, 2\}$, infinite. (d) $\varnothing$, finite.

SOLUTION

(a) A number less than every element of A must be less than the smallest, 3. The whole numbers less than 3 are 0, 1, 2, so the set is $\{0, 1, 2\}$, finite with 3 elements.

(b) A number greater than every element must be greater than the largest, 20. The set is $\{21, 22, 23, \dots\}$. It is infinite: adding 1 to any element gives another.

(c) Now negative numbers count too, and they run on forever downward: $\{\dots, 0, 1, 2\}$, infinite. Changing one word turned a finite set into an infinite one.

(d) Such a number would be greater than 20 and less than 3 at once, which no number is. The set is $\varnothing$, finite with 0 elements.

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