Average rate of change from a table or graph

A quantity rose fast, then slowly, then fell a little. What one number says how fast it changed between two chosen moments?

WORDS TO KNOW

Average rate of change: the change in output divided by the change in input, between two chosen inputs.

Interval: the stretch of inputs from a to b.

Units of a rate: output units **per** input unit, such as meters per second.

Δ (delta): "change in". Δy is the change in y, and Δx is the change in x.

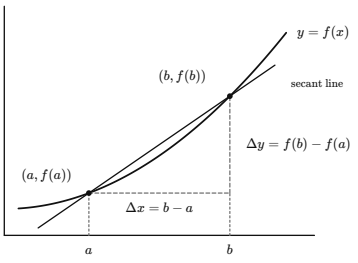
Secant line: a straight line through two points of a graph.

KEY IDEAS

1. The formula

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

End value minus start value, over end input minus start input. Subtract in the **same order** on top and bottom.



In a table, the two inputs pick out two rows. On a graph, they pick out two points. Everything in between is ignored.

2. It is the slope of the secant line

The average rate of change from a to b is the slope of the secant line through $(a, f(a))$ and $(b, f(b))$. A steeper secant line means a faster average change. A secant line falling to the right means a negative rate.

3. What the number means

It is the **one steady rate** that would give the same total change over the same interval.

Positive: the output ended higher. Negative: it ended lower. Zero: it ended where it started (not the same as staying put).

4. Lines give one answer; curves give many

For a line $y = mx + c$, every interval gives the slope m. For a curve, the answer depends on the interval, so always say which interval. Two different averages prove a function is not linear. Two equal averages do **not** prove it is.

WATCH OUT

- **Mixing the order.** $\frac{f(b) - f(a)}{a - b}$ flips the sign and turns a rise into a fall.
- **Reading it as the rate at one moment.** An average over 6 seconds is not the speed at second 3.
- **Thinking a positive average means it rose the whole time.** The quantity may have gone up and down in between.
- **Leaving off the units.** Say "people per year", not just a number.

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WORKED EXAMPLES

Example 1. Briony's distance $d(t)$ in a race, t seconds after the start. Find the average rate of change over the first two seconds and over all six seconds.

t (s)	0	1	2	3	4	5	6
$d(t)$ (m)	0	5	13	22	32	42	52

1. First two seconds, rows $t = 0$ and $t = 2$:
$$\frac{d(2) - d(0)}{2 - 0} = \frac{13 - 0}{2} = 6.5$$
2. All six seconds, rows $t = 0$ and $t = 6$:
$$\frac{d(6) - d(0)}{6 - 0} = \frac{52}{6} = \frac{26}{3} \approx 8.67$$

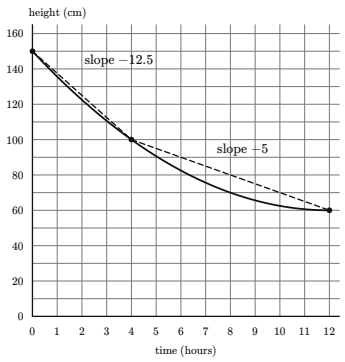
Answer: 6.5 meters per second, then about 8.67 meters per second. Different intervals give different answers, so d is not linear.

Example 2. A city had 180 thousand people in 1990 and 176 thousand in 2000. Find the average rate of change in people per year.

1. Later minus earlier, top and bottom:
$$\frac{176 - 180}{2000 - 1990} = \frac{-4}{10} = -0.4$$
2. That is −0.4 thousand people per year. Multiply by 1000: −400 people per year.

Answer: −400 people per year. On average the city lost 400 people a year.

Example 3. The graph shows a snowman's height $h(t)$, in cm, t hours after the sun came out. Compare the average rate of change from $t = 0$ to $t = 4$ with the one from $t = 4$ to $t = 12$.



1. Read the points: $h(0) = 150$, $h(4) = 100$, $h(12) = 60$.
2. First interval, in cm per hour:
$$\frac{100 - 150}{4 - 0} = \frac{-50}{4} = -12.5$$
3. Second interval, in cm per hour:
$$\frac{60 - 100}{12 - 4} = \frac{-40}{8} = -5$$

Answer: −12.5 and −5 cm per hour. The snowman melted faster at first, which shows as the steeper secant line.

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