Factoring $x^2 + bx + c$

Multiplying two binomials gives a trinomial like $x^2 + bx + c$. Given only the trinomial, how do you find the two binomials?

WORDS TO KNOW

Trinomial: a sum of three terms, such as $x^2 + 7x + 6$.

Factor pair of c: two integers that multiply to c, such as 3 and 8 for 24.

Prime over the integers: a trinomial that cannot be written as $(x + p)(x + q)$ with integers p and q.

Leading coefficient: the number in front of x^2. In this lesson it is 1.

Equivalent expressions: two expressions with the same value for every x.

KEY IDEAS

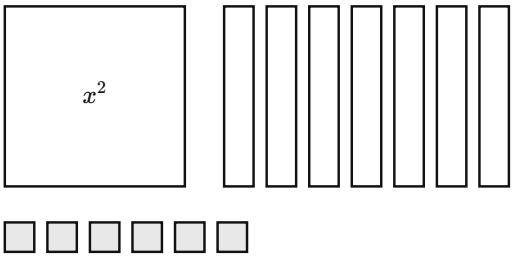
1. The middle number is a sum, the last number is a product

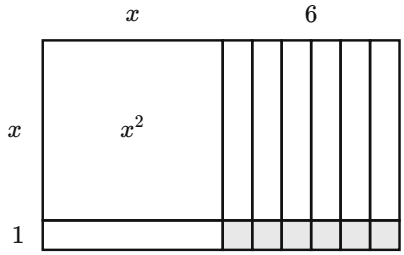
Multiply out $(x + p)(x + q)$: you get $x^2 + qx + px + pq = x^2 + (p + q)x + pq$. In the tiles, $x^2 + 7x + 6$ makes a rectangle $x + 6$ by $x + 1$, because 6 + 1 = 7 and 6 · 1 = 6.

To factor $x^2 + bx + c$, find two numbers p and q with

$$p + q = b \qquad \text{and} \qquad pq = c$$

Then $x^2 + bx + c = (x + p)(x + q)$. Check by multiplying back.





2. Search the products, and read the signs first

Only a few pairs multiply to c, so list those, not the pairs that add to b. Start at 1 and stop once the smaller number passes the square root of the size of c.

	Signs of the two numbers	What to look for
$c > 0$	same, both with the sign of b	a pair that **adds** to the size of b
$c < 0$	opposite	a pair whose sizes **differ** by the size of b; the larger takes the sign of b

If the full list has no pair that works, the trinomial is prime over the integers.

3. Two variables

$$x^2 + bxy + cy^2 = (x + py)(x + qy)$$

with the same two numbers p and q, each followed by y.

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WATCH OUT

- **Put the terms in order first:** x^2, then x, then the number. Only then read b and c. No x term means $b = 0$.
- **Give the minus sign to the right number.** $x^2 - 12x - 13 = (x - 13)(x + 1)$, not $(x + 13)(x - 1)$, which multiplies out to $x^2 + 12x - 13$.
- **Prime needs the whole list.** Skipping a factor pair can make a trinomial look prime when it is not.
- **The leading coefficient must be 1.** For a trinomial like $3x^2 + 8x + 5$ this method does not work as it is.

WORKED EXAMPLES

Example 1. Factor $x^2 - 18x + 65$.

$c = 65$ is positive and $b = -18$ is negative, so both numbers are negative. List the factor pairs of 65:

Factors of 65	Sum
−1, −65	−66
−5, −13	−18

1. The pair −5 and −13 multiplies to 65 and adds to −18.
2. So the binomials are $x - 5$ and $x - 13$.

Answer: $(x - 5)(x - 13)$. **Check:** $x^2 - 13x - 5x + 65 = x^2 - 18x + 65$.

Example 2. Factor $x^2 - 11x - 26$.

$c = -26$ is negative, so the numbers have opposite signs, and their sizes must differ by 11.

Factors of 26	Difference
1, 26	25
2, 13	11

1. The pair 2 and 13 differs by 11.
2. $b = -11$ is negative, so the larger size, 13, gets the minus sign: the numbers are −13 and 2.

Answer: $(x - 13)(x + 2)$. **Check:** $x^2 + 2x - 13x - 26 = x^2 - 11x - 26$.

Example 3. Factor $x^2 - 3x - 14$, or show it is prime over the integers.

$c = -14$ is negative, so the signs are opposite and the sizes must differ by 3.

Factors of 14	Difference
1, 14	13
2, 7	5

1. No pair differs by 3.
2. Every pair of integers that multiplies to −14 is on this list, with a minus on one number, so none was missed.

Answer: $x^2 - 3x - 14$ is prime over the integers. It still has a value for every x. It just has no factored form with integer numbers.

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