Algebra 1 Unit 8 worksheet: Factoring when the leading coefficient is one (answer key)

ANSWERS AT A GLANCE

1. (a) $(x+4)(x+5)$ (b) $(k-4)(k-9)$ (c) $(x+8)(x-6)$ (d) $(w-12)(w+5)$ (e) $(x-4)(x-16)$ (f) $(p+9)(p-8)$
2. (a) $(x+8)(x-3)$ (b) $(x-3y)(x-5y)$ (c) prime over the integers (d) prime over the integers (e) $(a-9b)(a+6b)$
3. (a) $x+13$ feet (b) 6 feet by 16 feet, area 96 square feet
4. (a) $x^2+2x-35$ (b) The larger number must take the sign of $b=-2$; $x^2-2x-35=(x-7)(x+5)$.
5. $c=7$, 12, 15 or 16
6. (a) $(x+3)(x+4)$ (b) 9900
7. (a) $pq=c$ is odd, so p and q are both odd, and $b=p+q$ is even. (b) It does not: 15 is odd and 7 is odd, so it is prime over the integers.
8. (a) 40 and 5 (b) 15 and 9 (c) 9 and 5

WORKED SOLUTIONS

1. Factor. Check each answer by multiplying back.

(a) $x^2+9x+20$ (b) $k^2-13k+36$ (c) $x^2+2x-48$ (d) $w^2-7w-60$ (e) $x^2-20x+64$ (f) p^2+p-72

ANSWER

(a) $(x+4)(x+5)$ (b) $(k-4)(k-9)$ (c) $(x+8)(x-6)$ (d) $(w-12)(w+5)$ (e) $(x-4)(x-16)$ (f) $(p+9)(p-8)$

SOLUTION

(a) $c=20$ and $b=9$ are both positive, so both numbers are positive. The pairs of 20 are 1, 20; 2, 10; 4, 5, and 4 + 5 = 9. So $(x+4)(x+5)$. Check: $x^2+5x+4x+20$.

(b) $c=36$ is positive and $b=-13$ is negative, so both numbers are negative. The pairs of 36 are 1, 36; 2, 18; 3, 12; 4, 9; 6, 6, and only 4 + 9 = 13. So $(k-4)(k-9)$. Check: $k^2-9k-4k+36$.

(c) $c=-48$, so the signs are opposite and the sizes differ by 2: the pair 6, 8. The larger takes the sign of $b=2$: the numbers are 8 and −6. So $(x+8)(x-6)$. Check: $x^2-6x+8x-48$.

(d) $c=-60$, so the sizes differ by 7: of 1, 60; 2, 30; 3, 20; 4, 15; 5, 12; 6, 10, only 5, 12 works. The larger is negative since $b=-7$: the numbers are −12 and 5. So $(w-12)(w+5)$. Check: $w^2+5w-12w-60$.

(e) $c=64$ is positive and $b=-20$ is negative, so both numbers are negative with sizes adding to 20. The pairs of 64 are 1, 64; 2, 32; 4, 16; 8, 8, and 4 + 16 = 20. So $(x-4)(x-16)$. Check: $x^2-16x-4x+64$.

(f) Here $b=1$ and $c=-72$, so the sizes differ by 1: the pair 8, 9, with the larger positive. The numbers are 9 and −8. So $(p+9)(p-8)$. Check: $p^2-8p+9p-72$.

2. Write the terms in order if needed, then factor. If a trinomial cannot be factored, say that it is prime over the integers and show why.

(a) $-24+x^2+5x$ (b) $x^2-8xy+15y^2$ (c) x^2+7x+8 (d) $x^2-2x-30$ (e) $a^2-3ab-54b^2$

ANSWER

(a) $(x+8)(x-3)$ (b) $(x-3y)(x-5y)$ (c) prime over the integers (d) prime over the integers (e) $(a-9b)(a+6b)$

SOLUTION

(a) In order: $x^2+5x-24$. The signs are opposite and the sizes differ by 5: of 1, 24; 2, 12; 3, 8; 4, 6, only 3, 8 works, and the larger is positive. So $(x+8)(x-3)$. Check: $x^2-3x+8x-24$.

Free sample from math.wiki for classroom use. Do not post. math.wiki

(b) Look for $(x+py)(x+qy)$ with $p+q=-8$ and $pq=15$, both negative. The pairs of 15 are 1, 15 and 3, 5, and 3 + 5 = 8. So $(x-3y)(x-5y)$. Check: $x^2-5xy-3xy+15y^2$.

(c) Both numbers would be positive with product 8 and sum 7. The pairs of 8 are 1, 8 (sum 9) and 2, 4 (sum 6). Neither adds to 7, and these are all the pairs, so x^2+7x+8 is prime over the integers.

(d) The signs would be opposite with sizes differing by 2. The pairs of 30 are 1, 30; 2, 15; 3, 10; 5, 6, with differences 29, 13, 7, 1. None is 2, so $x^2-2x-30$ is prime over the integers.

(e) Opposite signs, sizes differing by 3, larger negative. Of the pairs of 54 (1, 54; 2, 27; 3, 18; 6, 9), only 6, 9 differ by 3, so the numbers are −9 and 6. So $(a-9b)(a+6b)$. Check: $a^2+6ab-9ab-54b^2$.

3. A rectangular rug for a classroom reading corner has area $x^2+16x+39$ square feet. One side of the rug is $(x+3)$ feet long.

(a) Find the other side.

(b) Find both sides and the area of the rug when $x=3$.

ANSWER

(a) $x+13$ feet (b) 6 feet by 16 feet, area 96 square feet

SOLUTION

(a) Call the other side $x+q$. Then $(x+3)(x+q)=x^2+(3+q)x+3q$, so $3q=39$, which gives $q=13$. The middle coefficient agrees: 3 + 13 = 16. The other side is $x+13$ feet.

Check: $(x+3)(x+13)=x^2+13x+3x+39$, which is $x^2+16x+39$.

(b) When $x=3$ the sides are 3 + 3 = 6 feet and 3 + 13 = 16 feet, so the area is $6\cdot16=96$ square feet. The trinomial agrees: 9 + 48 + 39 = 96.

4. Diego factored $x^2-2x-35$ and wrote $(x+7)(x-5)$.

(a) Multiply out Diego's answer. Which trinomial did he actually factor?

(b) Explain his mistake and give the correct factorization.

ANSWER

(a) $x^2+2x-35$ (b) The larger number must take the sign of $b=-2$; $x^2-2x-35=(x-7)(x+5)$.

SOLUTION

(a) $(x+7)(x-5)=x^2-5x+7x-35=x^2+2x-35$. The middle term has the wrong sign.

(b) The constant −35 is negative, so the numbers have opposite signs and sizes that differ by 2: the pair 5, 7 is right. But their sum must be −2, so the larger size, 7, takes the sign of b and is negative. Diego gave the minus to the wrong number. The numbers are −7 and 5:

$$x^2-2x-35=(x-7)(x+5)$$

Check: $x^2+5x-7x-35=x^2-2x-35$.

5. Find every positive integer c for which x^2+8x+c factors as $(x+p)(x+q)$ with integers p and q. Explain how you know your list is complete.

ANSWER

$c=7$, 12, 15 or 16

SOLUTION

We need $p+q=8$ and $pq=c$. So $q=8-p$ and $c=p(8-p)$.

For c to be positive, p and q must have the same sign, and since they add to the positive number 8, both are positive. So p is one of 1, 2, ..., 7:

Free sample from math.wiki for classroom use. Do not post. math.wiki

$1 \cdot 7 = 7$, $2 \cdot 6 = 12$, $3 \cdot 5 = 15$, $4 \cdot 4 = 16$, and $p = 5, 6, 7$ give the same products again.

So $c = 7$, 12, 15 or 16. The list is complete because every pair of positive integers with sum 8 was tried.

6. (a) Factor $x^2 + 7x + 12$.

(b) Use your factored form to find the value of $x^2 + 7x + 12$ when $x = 96$, without a calculator.

ANSWER

(a) $(x + 3)(x + 4)$ (b) 9900

SOLUTION

(a) Both numbers are positive with product 12 and sum 7: 3 and 4. So $x^2 + 7x + 12 = (x + 3)(x + 4)$. Check: $x^2 + 4x + 3x + 12$.

(b) The two forms are equivalent, so they have the same value at $x = 96$:

$$(96 + 3)(96 + 4) = 99 \cdot 100 = 9900$$

(The long way agrees: 9216 + 672 + 12 = 9900.)

7. (a) Explain why, when c is an odd integer, $x^2 + bx + c$ can factor as $(x + p)(x + q)$ with integers p and q only if b is even.

(b) Use part (a) to decide, without listing factor pairs, whether $x^2 + 7x + 15$ factors over the integers.

ANSWER

(a) $pq = c$ is odd, so p and q are both odd, and $b = p + q$ is even. (b) It does not: 15 is odd and 7 is odd, so it is prime over the integers.

SOLUTION

(a) If $x^2 + bx + c = (x + p)(x + q)$, then $pq = c$ and $p + q = b$. An even number times any integer is even, so if p or q were even, c would be even. Since c is odd, p and q are both odd. The sum of two odd numbers is even, so $b = p + q$ is even.

(b) Here $c = 15$ is odd, so any factorization would need an even b. But $b = 7$ is odd, so $x^2 + 7x + 15$ is prime over the integers. (Listing confirms it: 1 + 15 = 16 and 3 + 5 = 8.)

8. Fill in each blank so that the factorization is true.

(a) $x^2 - 3x - \square = (x - 8)(x + \square)$

(b) $x^2 + \square\, x + 54 = (x + 6)(x + \square)$

(c) $x^2 + 4x - 45 = (x + \square)(x - \square)$

ANSWER

(a) 40 and 5 (b) 15 and 9 (c) 9 and 5

SOLUTION

(a) $(x - 8)(x + q)$ has middle coefficient $-8 + q$, so $-8 + q = -3$ and $q = 5$. The constant is $(-8)(5) = -40$, so the blank is 40: $x^2 - 3x - 40 = (x - 8)(x + 5)$.

(b) The constant $54 = 6q$, so $q = 9$, and the middle coefficient is 6 + 9 = 15: $x^2 + 15x + 54 = (x + 6)(x + 9)$.

(c) The numbers are p and $-r$ with p and r positive, $pr = 45$ and $p - r = 4$. Of the pairs 1, 45; 3, 15; 5, 9, only 9 − 5 = 4. So $x^2 + 4x - 45 = (x + 9)(x - 5)$.

Free sample from math.wiki for classroom use. Do not post. math.wiki