Using the quadratic formula

The quadratic formula solves every equation $ax^2 + bx + c = 0$. How do you use it without sign slips, and when is another method quicker?

WORDS TO KNOW

Standard form: $ax^2 + bx + c = 0$, with everything on one side and 0 on the other.

Discriminant: $b^2 - 4ac$, the number under the square root.

Coefficients a, b, c: the numbers in standard form, each with its sign. A missing term has coefficient 0.

Exact answer: an answer kept with its root, such as $3 + \sqrt{7}$, not a rounded decimal.

KEY IDEAS

1. Using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Put the equation in standard form.
2. Read off a, b, c, each with its sign.
3. Work out $b^2 - 4ac$ on its own first. Put negatives in parentheses.
4. Substitute, then split the $\pm$ into two answers.
5. Simplify the root. Cancel a factor only if it divides **every** term on top and the bottom.

The sign of $b^2 - 4ac$ tells you what is coming: positive gives two real solutions, zero gives one, negative gives none (the parabola never reaches the x-axis).

2. Choosing a method

Every correct method gives the same solutions. They differ only in effort, so look at the equation for a few seconds first.

What you see	Quickest method
no x term, or a squared binomial alone: $ax^2 = k$, $a(x - h)^2 = k$	square root property
no constant term: $ax^2 + bx = 0$	factor out x
factors easily over the integers	factoring
$a = 1$ and b even	completing the square (no fractions)
none of these	the quadratic formula (it always works)

The factoring test. When a, b, c are integers, the quadratic factors over the integers exactly when $b^2 - 4ac$ is a perfect square (0, 1, 4, 9, 16, ...).

WATCH OUT

- **Standard form first.** In $x^2 + 5x = 8$, c is not 8. Subtract first: $x^2 + 5x - 8 = 0$, so $c = -8$.
- **Parentheses around negatives.** With $b = -3$: $b^2 = (-3)^2 = 9$, and $-b = 3$.
- **Cancel from every term.** $\dfrac{6 \pm 3\sqrt{2}}{3} = 2 \pm \sqrt{2}$, not $2 \pm 3\sqrt{2}$.
- **Negative under the root** means no real solutions. Do not drop the minus sign to make it work.

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WORKED EXAMPLES

Example 1. Solve $3x^2 = 4x + 4$.

1. Standard form: subtract $4x$ and 4 from both sides: $3x^2 - 4x - 4 = 0$. So $a = 3$, $b = -4$, $c = -4$.
2. Discriminant: $(-4)^2 - 4(3)(-4) = 16 + 48 = 64$.
3. Substitute: $x = \dfrac{-(-4) \pm \sqrt{64}}{2(3)} = \dfrac{4 \pm 8}{6}$.
4. Split: $\dfrac{4+8}{6} = 2$ and $\dfrac{4-8}{6} = -\dfrac{2}{3}$.

Answer: $x = 2$ or $x = -\dfrac{2}{3}$. **Check:** 3(4) = 12 and 4(2) + 4 = 12. Since 64 is a perfect square, this one also factors: $(x - 2)(3x + 2)$.

Example 2. Solve $6x^2 + 2x - 1 = 0$. Give exact answers.

1. $a = 6$, $b = 2$, $c = -1$. Discriminant: $2^2 - 4(6)(-1) = 4 + 24 = 28$.
2. Simplify the root: $\sqrt{28} = \sqrt{4 \cdot 7} = 2\sqrt{7}$.
3. Substitute: $x = \dfrac{-2 \pm 2\sqrt{7}}{12}$.
4. Every term on top and the bottom share the factor 2. Divide each by 2.

Answer: $x = \dfrac{-1 \pm \sqrt{7}}{6}$, about 0.27 and −0.61. **Check:** the two add to $-\dfrac{2}{6} = -\dfrac{1}{3}$, which is $-\dfrac{b}{a}$.

Example 3. Choose a method and solve: (a) $5x^2 = 45$; (b) $x^2 + 9x = 0$; (c) $x^2 + 3x - 1 = 0$.

1. (a) No x term, so use square roots. Divide by 5: $x^2 = 9$, so $x = \pm 3$.
2. (b) No constant term, so factor out x: $x(x + 9) = 0$, so $x = 0$ or $x = -9$.
3. (c) The discriminant is 9 + 4 = 13, not a perfect square, so it does not factor. Use the formula: $x = \dfrac{-3 \pm \sqrt{13}}{2}$.

Answer: (a) $x = \pm 3$; (b) $x = 0$ or $x = -9$; (c) $x = \dfrac{-3 \pm \sqrt{13}}{2}$, about 0.30 and −3.30.

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