Algebra 1 Unit 11 worksheet: Using the quadratic formula (answer key)

ANSWERS AT A GLANCE

1. (a) $x = -2 \pm \sqrt{11}$ (b) $x = 2$ or $x = \frac{1}{3}$ (c) $y = \frac{-1 \pm \sqrt{7}}{2}$ (d) no real solutions (e) $x = 3 \pm \sqrt{5}$
2. (a) $x = \frac{1}{3}$ or $x = -2$ (b) $x = \frac{3 \pm \sqrt{41}}{4}$ (c) $x = \frac{-2 \pm \sqrt{10}}{3}$ (d) $x = \frac{-5 \pm \sqrt{17}}{2}$
3. (a) factoring (no constant term): $x = 0$ or $x = -\frac{2}{7}$ (b) the formula: $x = \frac{1 \pm \sqrt{61}}{10}$ (c) the square root property: $x = \frac{1}{2}$ or $x = -\frac{5}{2}$ (d) completing the square: $x = 6 \pm \sqrt{6}$
4. (a) He wrote $-b$ as −8, but $b = -8$, so $-b = -(-8) = 8$; and in the last step he divided only the first term of the numerator by 4. (b) $x = \frac{4 \pm 3\sqrt{2}}{2}$
5. 7 days; on the last day she did 10 push-ups.
6. (a) $x = 45 \pm 5\sqrt{5}$, about 56.18 cm and 33.82 cm. (b) Yes, both lie between 0 and 90. They are the two sticks of the same kite: if one stick is about 56.18 cm, the other is 90 − 56.18 = 33.82 cm, and the other way around. (c) $x^2 - 90x + 2100 = 0$ has $b^2 - 4ac = -300 < 0$.

WORKED SOLUTIONS

1. Solve with the quadratic formula. Give exact answers with radicals and fractions in simplest form. If an equation has no real solutions, say so.

(a) $x^2 + 4x - 7 = 0$ (b) $3x^2 - 7x + 2 = 0$ (c) $2y^2 + 2y - 3 = 0$

(d) $5m^2 - 3m + 1 = 0$ (e) $-x^2 + 6x - 4 = 0$

ANSWER

(a) $x = -2 \pm \sqrt{11}$ (b) $x = 2$ or $x = \frac{1}{3}$ (c) $y = \frac{-1 \pm \sqrt{7}}{2}$ (d) no real solutions (e) $x = 3 \pm \sqrt{5}$

SOLUTION

(a) $a = 1$, $b = 4$, $c = -7$. $b^2 - 4ac = 16 + 28 = 44$, and $\sqrt{44} = 2\sqrt{11}$. So $x = \frac{-4 \pm 2\sqrt{11}}{2} = -2 \pm \sqrt{11}$.

(b) $a = 3$, $b = -7$, $c = 2$. $b^2 - 4ac = 49 - 24 = 25$. So $x = \frac{7 \pm 5}{6}$, which gives $x = 2$ or $x = \frac{1}{3}$.

(c) $a = 2$, $b = 2$, $c = -3$. $b^2 - 4ac = 4 + 24 = 28$, and $\sqrt{28} = 2\sqrt{7}$. So $y = \frac{-2 \pm 2\sqrt{7}}{4}$. The 2 is a factor of every term of the numerator and of the 4, so $y = \frac{-1 \pm \sqrt{7}}{2}$.

(d) $a = 5$, $b = -3$, $c = 1$. $b^2 - 4ac = 9 - 20 = -11$. No real number squares to −11, so there are no real solutions.

(e) $a = -1$, $b = 6$, $c = -4$. $b^2 - 4ac = 36 - 4(-1)(-4) = 36 - 16 = 20$, and $\sqrt{20} = 2\sqrt{5}$. So $x = \frac{-6 \pm 2\sqrt{5}}{-2} = 3 \mp \sqrt{5}$, which is the same pair as $x = 3 \pm \sqrt{5}$.

2. Put each equation in standard form, clearing any decimals or fractions, then solve it with the quadratic formula. Give exact answers in simplest form.

(a) $3x^2 = 2 - 5x$ (b) $x(2x - 3) = 4$

(c) $0.3x^2 + 0.4x - 0.2 = 0$ (d) $2x(x + 3) = (x + 2)(x - 1)$

ANSWER

(a) $x = \frac{1}{3}$ or $x = -2$ (b) $x = \frac{3 \pm \sqrt{41}}{4}$ (c) $x = \frac{-2 \pm \sqrt{10}}{3}$ (d) $x = \frac{-5 \pm \sqrt{17}}{2}$

SOLUTION

(a) Add $5x$ and subtract 2: $3x^2 + 5x - 2 = 0$. $b^2 - 4ac = 25 + 24 = 49$, so $x = \frac{-5 \pm 7}{6}$: $x = \frac{1}{3}$ or $x = -2$.

Free sample from math.wiki for classroom use. Do not post. math.wiki

(b) Expand and subtract 4: $2x^2 - 3x - 4 = 0$. $b^2 - 4ac = 9 + 32 = 41$, which has no square factor, so $x = \dfrac{3 \pm \sqrt{41}}{4}$.
(c) Multiply both sides by 10: $3x^2 + 4x - 2 = 0$. $b^2 - 4ac = 16 + 24 = 40$, $\sqrt{40} = 2\sqrt{10}$, so $x = \dfrac{-4 \pm 2\sqrt{10}}{6} = \dfrac{-2 \pm \sqrt{10}}{3}$.
(d) Expand both sides: $2x^2 + 6x = x^2 + x - 2$, so $x^2 + 5x + 2 = 0$. $b^2 - 4ac = 25 - 8 = 17$, so $x = \dfrac{-5 \pm \sqrt{17}}{2}$.

3. For each equation, name the method that reaches the solutions in the fewest steps (factoring, the square root property, completing the square or the formula) and say what in the equation tells you. Then solve it.

(a) $7x^2 + 2x = 0$

(b) $5x^2 - x - 3 = 0$

(c) $4(x + 1)^2 = 9$

(d) $x^2 - 12x + 30 = 0$

ANSWER

(a) factoring (no constant term): $x = 0$ or $x = -\dfrac{2}{7}$ (b) the formula: $x = \dfrac{1 \pm \sqrt{61}}{10}$ (c) the square root property: $x = \dfrac{1}{2}$ or $x = -\dfrac{5}{2}$ (d) completing the square: $x = 6 \pm \sqrt{6}$

SOLUTION

(a) There is no constant term, so x is a common factor: $x(7x + 2) = 0$, giving $x = 0$ or $x = -\dfrac{2}{7}$. Do not divide by x: that loses $x = 0$.
(b) $b^2 - 4ac = 1 + 60 = 61$ is not a perfect square, so it does not factor over the integers, and $a = 5$ makes completing the square messy. The formula: $x = \dfrac{1 \pm \sqrt{61}}{10}$.
(c) The variable appears only inside the squared binomial. Divide by 4: $(x + 1)^2 = \dfrac{9}{4}$, so $x + 1 = \pm\dfrac{3}{2}$ and $x = \dfrac{1}{2}$ or $x = -\dfrac{5}{2}$.
(d) $b^2 - 4ac = 144 - 120 = 24$ is not a perfect square, so factoring is out; the coefficient of x^2 is 1 and the coefficient of x is even, so completing the square is short. $x^2 - 12x = -30$, add 36: $(x - 6)^2 = 6$, so $x = 6 \pm \sqrt{6}$.

4. Kofi solved $2x^2 - 8x - 1 = 0$ like this:

$$a = 2, \quad b = -8, \quad c = -1$$

$$x = \frac{-8 \pm \sqrt{64 + 8}}{4} = \frac{-8 \pm \sqrt{72}}{4} = \frac{-8 \pm 6\sqrt{2}}{4} = -2 \pm 6\sqrt{2}$$

(a) Kofi made two mistakes, one when he substituted into the formula and one in the last step. Name them.
(b) Solve the equation correctly. Give exact answers in simplest form.

ANSWER

(a) He wrote $-b$ as −8, but $b = -8$, so $-b = -(-8) = 8$; and in the last step he divided only the first term of the numerator by 4. (b) $x = \dfrac{4 \pm 3\sqrt{2}}{2}$

SOLUTION

Free sample from math.wiki for classroom use. Do not post. math.wiki

(a) The formula starts with $-b$, the opposite of b. With $b = -8$ that is −(−8) = 8, not −8. (His number under the root is right: $(-8)^2 = 64$ and −4(2)(−1) = 8.) In the last step, the fraction bar groups the whole numerator, so the 4 underneath must divide both −8 and $6\sqrt{2}$; Kofi divided only the first term. (Correctly, his own expression would be $\dfrac{-8 \pm 6\sqrt{2}}{4} = \dfrac{-4 \pm 3\sqrt{2}}{2}$.)

(b) With $a = 2$, $b = -8$, $c = -1$:

$$b^2 - 4ac = 64 + 8 = 72$$

and $\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$. So

$$x = \frac{8 \pm 6\sqrt{2}}{4} = \frac{2(4 \pm 3\sqrt{2})}{2 \cdot 2} = \frac{4 \pm 3\sqrt{2}}{2}$$

5. Maya starts a push-up plan. On day 1 she does 4 push-ups, and on each day after that she does one more push-up than the day before. After n days her total is $\dfrac{n(n+7)}{2}$ push-ups. When the plan ends, her total was 49 push-ups. Use the quadratic formula to find how many days the plan lasted, then find how many push-ups she did on the last day.

ANSWER

7 days; on the last day she did 10 push-ups.

SOLUTION

Set the total equal to 49 and multiply both sides by 2:

$$n(n+7) = 98 \qquad \text{so} \qquad n^2 + 7n - 98 = 0$$

Here $a = 1$, $b = 7$, $c = -98$:

$$b^2 - 4ac = 7^2 - 4(1)(-98) = 49 + 392 = 441$$

and $\sqrt{441} = 21$. So

$$n = \frac{-7 \pm 21}{2}$$

The plus sign gives $n = \dfrac{14}{2} = 7$; the minus sign gives $n = -14$, and a number of days cannot be negative. So the plan lasted 7 days.

The daily counts run 4, 4 + 1, ..., so on day 7 she did 4 + 7 − 1 = 10 push-ups. Check: $\dfrac{7(7+7)}{2} = 49$.

6. A kite is made from a 90 cm wooden dowel cut into two sticks. The sticks are fastened across each other at right angles and form the kite's two diagonals, and the kite's area is half the product of the two sticks' lengths. Let x be the length of one stick, in cm.

(a) Find every value of x that gives a kite of area 950 cm^2, exactly and to two decimal places.

(b) Do both values make a real kite? Explain how the two values are related.

(c) Use $b^2 - 4ac$ to show that no cut gives a kite of area 1050 cm^2.

ANSWER

(a) $x = 45 \pm 5\sqrt{5}$, about 56.18 cm and 33.82 cm. (b) Yes, both lie between 0 and 90. They are the two sticks of the same kite: if one stick is about 56.18 cm, the other is 90 − 56.18 = 33.82 cm, and the other

Free sample from math.wiki for classroom use. Do not post. math.wiki

way around. (c) $x^2 - 90x + 2100 = 0$ has $b^2 - 4ac = -300 < 0$.

SOLUTION

(a) The other stick is $90 - x$ cm long, so the area is $\frac{x(90-x)}{2} = 950$. Multiply by 2: $x(90-x) = 1900$, so $90x - x^2 = 1900$. Move everything to one side: $x^2 - 90x + 1900 = 0$. Then $a = 1$, $b = -90$, $c = 1900$:

$$b^2 - 4ac = 8100 - 7600 = 500$$

and $\sqrt{500} = \sqrt{100 \cdot 5} = 10\sqrt{5}$, so

$$x = \frac{90 \pm 10\sqrt{5}}{2} = 45 \pm 5\sqrt{5}$$

With $5\sqrt{5} \approx 11.180$: $x \approx 56.18$ or $x \approx 33.82$.

(b) Each stick must have positive length, so $0 < x < 90$. Both values fit. The two solutions add to 90: when $x \approx 56.18$ the other stick is $90 - x \approx 33.82$, and when $x \approx 33.82$ the other stick is about 56.18. So there is really one kite, and the two solutions are its two sticks, depending on which stick x names.

(c) $\frac{x(90-x)}{2} = 1050$ gives $x(90-x) = 2100$, or $x^2 - 90x + 2100 = 0$. Then $b^2 - 4ac = 8100 - 8400 = -300$, which is negative, so there is no real x: no cut gives that area.

Free sample from math.wiki for classroom use. Do not post. math.wiki