Domain and range from a graph

A graph shows a function as a picture. Which inputs does it use, and which outputs does it ever reach?

WORDS TO KNOW

Domain: the inputs (x-values) a function uses.

Range: the outputs (y-values) it actually reaches.

Closed dot: the point is on the graph.

Open dot: the point is **not** on the graph.

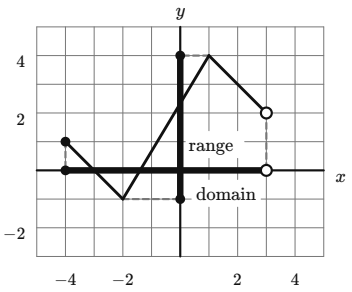
Arrow: the graph goes on forever in that direction.

Interval notation: $[a, b)$ means from a, included, up to b, not included. A bracket includes an end; a parenthesis leaves it out. $\cup$ ("or") joins two pieces.

KEY IDEAS

1. Domain and range are shadows

Shine a light straight down on the graph: its shadow on the x-axis is the domain. Shine a light from the side: its shadow on the y-axis is the range.



Domain: from the leftmost point to the rightmost point.

Range: from the lowest point to the highest point, which may be in the middle of the graph, not at its ends.

Write each from smaller to bigger. An end at ∞ or $-\infty$ always gets a parenthesis.

2. Dots and arrows

- A closed dot gives a bracket. An open dot gives a parenthesis, **unless another point of the graph reaches that number anyway**.
- An arrow to the right gives ∞ in the domain; an arrow down gives $-\infty$ in the range.

3. Pieces, holes and leveling off

Two pieces: each piece throws its own shadow. Join them with $\cup$ where there is a gap. Shadows that overlap merge.

A hole: it always removes its input. It removes its output only if no other point has that height.

Leveling off: a graph that gets closer and closer to a height but never reaches it leaves that height out, with a parenthesis, even with no open dot drawn.

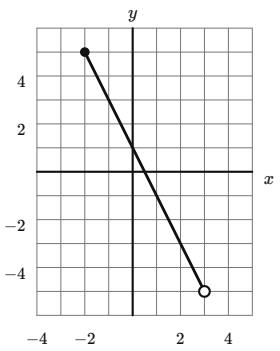
WATCH OUT

- **The ends of the range are not always the ends of the graph.** Look for the lowest and highest points, including turning points.
- **An open dot does not always remove its y-value.** Check whether the graph passes that height somewhere else.
- **The range does not start at 0 just because the axis does.**
- **A flat line has a one-number range,** like $\{-2\}$, not an interval.

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WORKED EXAMPLES

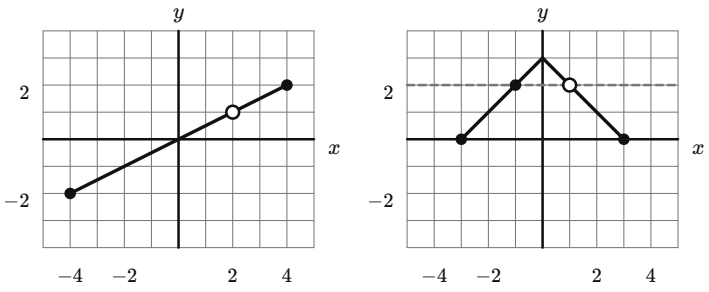
Example 1. Find the domain and range of the segment from the closed dot (−2, 5) to the open dot (3, −5).



1. Domain: the left end $x = -2$ is closed, so it is in. The right end $x = 3$ is open, so it is out. So $-2 \le x < 3$.
2. Range: the segment falls. The highest point is (−2, 5), included. The lowest would be height −5, but that point is missing, and no other point is that low.
3. So $-5 < y \le 5$.

Answer: domain $[-2, 3)$, range $(-5, 5]$. Each bracket comes from the dot at its own end.

Example 2. Find the domain and range of each graph. (a) The segment from the closed dot (−4, −2) to the closed dot (4, 2), with (2, 1) missing. (b) The tent from (−3, 0) up to (0, 3) and down to (3, 0), all closed, with (1, 2) missing.



1. (a) The hole removes $x = 2$: domain $[-4, 2) \cup (2, 4]$.
2. (a) The segment rises steadily, so height 1 is reached only at the missing point. Range $[-2, 1) \cup (1, 2]$.
3. (b) The hole removes $x = 1$: domain $[-3, 1) \cup (1, 3]$.
4. (b) The missing point has height 2, but the rising side passes height 2 at (−1, 2). So 2 is still reached. Lowest 0, highest 3: range $[0, 3]$.

Answer: (a) domain $[-4, 2) \cup (2, 4]$, range $[-2, 1) \cup (1, 2]$; (b) domain $[-3, 1) \cup (1, 3]$, range $[0, 3]$.

Example 3. Tea at 80°C cools toward room temperature, 24°C, and never reaches it. The graph of its temperature T after t minutes starts at the closed dot (0, 80), falls more and more slowly toward the level $T = 24$, and has an arrow to the right. Find the domain and range.

1. Domain: starts at $t = 0$, included, and the arrow goes on forever: $t \ge 0$.
2. Range: the highest point is the start, 80, included. The graph comes as close to 24 as you like but never touches it, so 24 is left out.

Answer: domain $[0, \infty)$, range $(24, 80]$.

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