The square root and reciprocal parent functions

The graph of $y = \sqrt{x}$ starts at a point and only goes right. The graph of $y = \dfrac{1}{x}$ comes in two pieces and never touches the axes. Why, and what are their domains and ranges?

WORDS TO KNOW

Square root function: $f(x) = \sqrt{x}$. For $x \geq 0$, $\sqrt{x}$ is the one number that is at least 0 and squares to x.

Endpoint: the point where a graph starts. The square root graph starts at (0, 0).

Domain and **range:** all the inputs a function accepts, and all the outputs it gives.

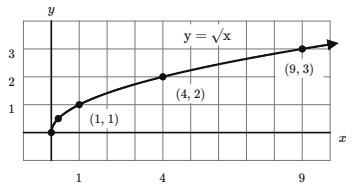
Reciprocal function: $g(x) = \dfrac{1}{x}$, the number that gives 1 when multiplied by x. It needs $x \neq 0$.

Asymptote: a line the graph gets as close to as you like, without end, as it runs along it.

KEY IDEAS

1. The square root graph is half a parabola on its side

On $y = x^2$ the point (3, 9) says 3 squares to 9. The square root reads that fact backwards, so its graph holds (9, 3). Swapping the coordinates of the right half of the U gives the square root graph. Only the right half, because $\sqrt{x}$ is never negative.



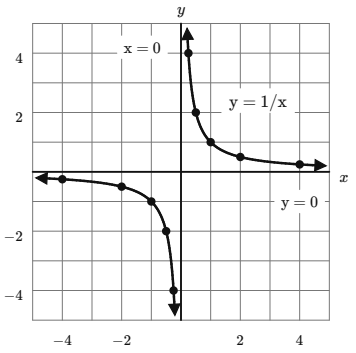
$y = \sqrt{x}$ starts at (0, 0) and passes through (1, 1), (4, 2), (9, 3).

Domain $x \geq 0$, range $y \geq 0$. It always rises, but more and more slowly: to climb from height n to $n + 1$ the input must grow by $2n + 1$.

For $0 < x < 1$, $\sqrt{x} > x$. For $x > 1$, $\sqrt{x} < x$.

2. The reciprocal graph trades big for small

The coordinates of every point multiply to 1. So as one gets close to 0, the other must grow. Near the y-axis the graph shoots up. Far to the right it sinks toward the x-axis, but it reaches neither axis.



$y = \dfrac{1}{x}$ has two pieces, in the first and third quadrants, through (1, 1) and (−1, −1).

Domain $x \neq 0$, range $y \neq 0$. Asymptotes: the lines $x = 0$ and $y = 0$, which are the two axes.

Each piece falls from left to right. If (a, b) is on the graph, so are (b, a) and $(-a, -b)$.

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3. The six parents side by side

Parent	Domain	Range	Shape
$y = x$	all	all	line, slope 1
$y = x^2$	all	$y \geq 0$	U, lowest at (0, 0)
$y = x^3$	all	all	S, flat at (0, 0)
$y = \lvert x \rvert$	all	$y \geq 0$	V, corner at (0, 0)
$y = \sqrt{x}$	$x \geq 0$	$y \geq 0$	half curve from (0, 0)
$y = \dfrac{1}{x}$	$x \neq 0$	$y \neq 0$	two pieces

Every one passes through (1, 1), so that point never tells two parents apart. Use other inputs.

WATCH OUT

- **$\sqrt{x}$ is never negative.** $x^2 = 9$ has two answers, but $\sqrt{9}$ is 3 alone. An equation like $\sqrt{x} = -4$ has no solution; squaring would suggest $x = 16$, and the check $\sqrt{16} = 4$ catches the mistake.
- **$\dfrac{1}{x}$ is never 0.** If $\dfrac{1}{x} = 0$, multiplying by x gives 1 = 0. So the graph never touches the x-axis, and it has no point at $x = 0$.
- **The reciprocal falls on each piece, not across the gap.** Every negative input gives a negative output, below the output of any positive input.
- **Multiplying an inequality by a negative x flips it.** For $x < 0$, treat the inequality as a new case.

WORKED EXAMPLES

Example 1. Find every x with $\sqrt{x} = 11$, with $\sqrt{x} = -5$, with $\dfrac{1}{x} = 20$ and with $\dfrac{1}{x} = -1.25$.

1. Square both sides of $\sqrt{x} = 11$: $x = 121$. Check: $\sqrt{121} = 11$.
2. $\sqrt{x} = -5$ has no solution, because the range of $\sqrt{x}$ is $y \geq 0$.
3. Multiply $\dfrac{1}{x} = 20$ by x (allowed, since $x \neq 0$): $1 = 20x$, so $x = 0.05$.
4. In the same way $\dfrac{1}{x} = -1.25$ gives $1 = -1.25x$, so $x = -0.8$.

Answer: $x = 121$; no solution; $x = 0.05$; $x = -0.8$. **Check:** $\dfrac{1}{0.05} = 20$ and $\dfrac{1}{-0.8} = -1.25$.

Example 2. Find every $x > 0$ with $\dfrac{1}{x} > 400$, every $x > 0$ with $\dfrac{1}{x} < 0.0025$, and every $x < 0$ with $\dfrac{1}{x} < -400$.

1. For $x > 0$, multiplying by x keeps the direction. $\dfrac{1}{x} > 400$ gives $1 > 400x$, so $x < 0.0025$. With $x > 0$: $0 < x < 0.0025$.
2. $\dfrac{1}{x} < 0.0025$ gives $1 < 0.0025x$, so $x > 400$.
3. For $x < 0$, multiplying by x flips the direction. $\dfrac{1}{x} < -400$ gives $1 > -400x$, so $x > -0.0025$. With $x < 0$: $-0.0025 < x < 0$.

Answer: $0 < x < 0.0025$; $x > 400$; $-0.0025 < x < 0$. **Check:** $x = 0.002$ gives $\dfrac{1}{x} = 500 > 400$, and $x = -0.002$ gives −500 < −400.

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