Symmetry about the y-axis and the origin

Some graphs fold onto themselves along the y-axis. Others land on themselves after a half turn about the origin. How do you spot each one, and what does it tell you about the half you cannot see?

WORDS TO KNOW

Symmetric about the vertical axis: folding the page along the y-axis lays the graph exactly onto itself.

Symmetric about the origin: turning the page half a turn (180°) about the origin lays the graph exactly onto itself.

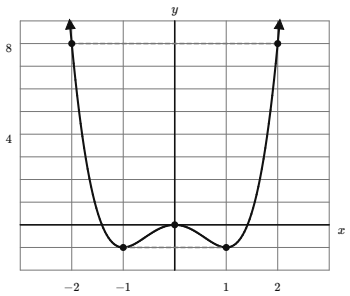
Partner: the point a given point must be matched with for the symmetry to hold.

Turning point: a peak (relative maximum) or a dip (relative minimum) of the graph.

KEY IDEAS

1. Each symmetry pairs every point with a partner

Symmetry is a claim about **every** point: each one needs its partner on the graph.



> Vertical axis: (a, b) pairs with $(-a, b)$, the same height on the other side.
>
> Origin: (a, b) pairs with $(-a, -b)$, as far out on the far side of the origin.

A half turn is two folds in a row: $(a, b) \mapsto (-a, b) \mapsto (-a, -b)$.

2. What each symmetry forces

	Vertical axis	Origin
x-intercepts	in pairs a and $-a$	in pairs a and $-a$
a peak's partner	a peak	a dip
rising on the right means, on the left	falling	rising
the two ends	head the same way	head opposite ways
a function's point at $x = 0$, if it has one	any height	(0, 0)

3. The parent graphs

Parent	Symmetric about	A pair of partners
$y = x^2$ and $y = \lvert x \rvert$	the vertical axis	(0.5, 0.25) and (−0.5, 0.25) on $y = x^2$
$y = x$, $y = x^3$, $y = \dfrac{1}{x}$	the origin	(0.5, 0.125) and (−0.5, −0.125) on $y = x^3$
$y = \sqrt{x}$	neither	nothing lies left of the axis

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WATCH OUT

- **One missing partner rules a symmetry out.** Matching pairs in a picture only make it believable; a picture shows just a window.
- **A function symmetric about the origin passes through it, if it has a point at $x = 0$.** A point $(0, c)$ would need the partner $(0, -c)$, and a function has one output at $x = 0$, so $c = 0$. ($y = 1/x$ has no point at $x = 0$.)
- **The mirror is the y-axis and the center is the origin.** $y = (x - 6)^2$ is a mirror image about the line $x = 6$, but it does not have the symmetry of this lesson.
- **Symmetry is not a flip.** A reflection makes a new graph; a symmetry is a move that leaves the graph as it was.

WORKED EXAMPLES

Example 1. The right half of a graph is a curve for x from 1 to 5 through (1, 5), (3, −2) and (5, 0). Complete it (a) so the graph is symmetric about the vertical axis, (b) so it is symmetric about the origin.

1. (a) Each (a, b) gets the partner $(-a, b)$: (−1, 5), (−3, −2) and (−5, 0).
2. (b) Each (a, b) gets the partner $(-a, -b)$: (−1, −5), (−3, 2) and (−5, 0).
3. (−5, 0) is in both answers: the partner of an x-intercept is an x-intercept either way, since −0 = 0.

Answer: (a) (−1, 5), (−3, −2), (−5, 0). (b) (−1, −5), (−3, 2), (−5, 0). Both are functions, since all the new points have x between −5 and −1, where nothing was drawn before.

Example 2. Is the graph of $y = x^3 - 4x$ symmetric about the vertical axis, the origin, both or neither?

1. Find some points: $x = 1$ gives $1 - 4 = -3$, and $x = -1$ gives $-1 + 4 = 3$. At $x = 3$: $27 - 12 = 15$; at $x = -3$: $-27 + 12 = -15$.
2. (1, −3) and (−1, 3) are half-turn partners, and so are (3, 15) and (−3, −15). The graph passes through (0, 0). So it looks symmetric about the origin.
3. Vertical axis: the mirror partner of (1, −3) would be (−1, −3), but the height at $x = -1$ is 3. One missing partner rules it out.

Answer: symmetric about the origin only. **Check:** the x-intercepts pair up: $x^3 - 4x = x(x - 2)(x + 2)$ is 0 at $x = -2$, 0 and 2.

Example 3. A function's graph is symmetric about the vertical axis. For $x \geq 0$ it has a relative maximum at (2, 6) and its only x-intercept at $x = 4$. It rises from $x = 0$ to $x = 2$ and falls for $x > 2$. Describe the left half.

1. The partner of the peak (2, 6) is a peak at (−2, 6): a fold keeps a peak a peak.
2. The partner of the intercept $x = 4$ is $x = -4$. So there are two x-intercepts.
3. A fold turns rising into falling: the graph falls from $x = -2$ to $x = 0$, and rises for $x < -2$.
4. The right end goes down, and a fold keeps both ends heading the same way, so the left end goes down too.

Answer: a peak at (−2, 6); x-intercepts −4 and 4; rising for $x < -2$ and falling from −2 to 0; both ends go down. The point at $x = 0$ is a dip, its own partner.

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