Shifting a graph up, down, left and right

Add a number to a function's output, or to its input, and the graph slides to a new place with its shape unchanged. Which way does it slide, and how far?

WORDS TO KNOW

Translation: a slide of the whole graph, with no change of shape.

Horizontal shift: a slide straight left or right.

Asymptote: a line the graph gets closer and closer to without end. For $y = \frac{1}{x}$ these are $x = 0$ and $y = 0$.

Vertical shift: a slide straight up or down.

Anchor: the one point a parent graph is known by, such as the vertex of $y = x^2$. You follow it to see where the graph went.

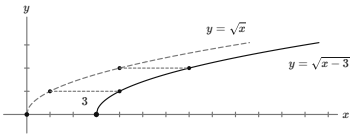
KEY IDEAS

1. A number added to the output moves the graph up or down

In $g(x) = f(x) + k$, the function does what f does and then adds k. So at every input the graph of g is k higher. The input does not change, so nothing moves sideways.

> $g(x) = f(x) + k$ moves every point (a, b) to $(a, b + k)$.
>
> $k > 0$: up k. $k < 0$: down $|k|$.

2. A number inside moves the graph sideways, the opposite way to its sign



In $g(x) = f(x - h)$, the function subtracts h before f works. To hand f the same number as before, the input must be h larger. So every height arrives h later along the x-axis.

> $g(x) = f(x - h)$ moves every point (a, b) to $(a + h, b)$.
>
> $f(x - 3)$ moves 3 **right**. $f(x + 3) = f(x - (-3))$ moves 3 **left**.

3. Both moves at once, read from the anchor

> $g(x) = f(x - h) + k$ moves every point (a, b) to $(a + h, b + k)$: h across and k up. The order of the two slides does not matter.

Parent	Anchor
$y = x^2$	the vertex (0, 0)
$y = x^3$	the center (0, 0), where it flattens
$y = \sqrt{x}$	the starting point (0, 0)
$y = \frac{1}{x}$	the point (0, 0) where the asymptotes cross

If the anchor lands on (h, k), the rule is $y = f(x - h) + k$. Then test one more point: the parent's (1, 1) must land on $(1 + h, 1 + k)$. If it does not, the shape changed, and a slide alone cannot explain the graph.

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WATCH OUT

- **A minus sign inside moves the graph right.** It does not mean "move left". Read the inside as $x - h$ first, then h is the shift.
- **Inside and outside are different moves.** $(x-2)^2$ slides the parabola right 2; $x^2 - 2$ slides it down 2.
- **x must stand alone inside.** $\sqrt{2x-6}$ is not $\sqrt{x}$ moved 6 right. It is $\sqrt{2(x-3)}$, a squeeze and then a shift.
- **Asymptotes and domains move too.** A horizontal shift moves the domain and the vertical asymptote. A vertical shift moves the range and the horizontal asymptote.

WORKED EXAMPLES

Example 1. Describe the graph of $g(x) = \sqrt{x+8} + 1$ compared with $y = \sqrt{x}$. Give its starting point, domain and range.

1. Write the inside as $x - h$: $x + 8 = x - (-8)$, so $h = -8$. The number outside is $k = 1$.
2. So the graph moves 8 left and 1 up. The starting point (0, 0) lands on (−8, 1).
3. The inside $x + 8$ must not be negative, so $x \geq -8$. The outputs start at 1 and grow, so $y \geq 1$.

Answer: 8 left and 1 up; starting point (−8, 1); domain $x \geq -8$; range $y \geq 1$. **Check:** the parent's (4, 2) should land on (−4, 3), and $\sqrt{-4+8} + 1 = 2 + 1 = 3$.

Example 2. Describe the graph of $g(x) = \dfrac{1}{x-7} + 2$ compared with $y = \dfrac{1}{x}$, including its asymptotes.

1. Here $h = 7$ and $k = 2$, so every point moves 7 right and 2 up.
2. Vertical asymptote: g cannot use the input where $x - 7 = 0$, so it is $x = 7$.
3. Horizontal asymptote: $\dfrac{1}{x-7}$ is never 0, so g never takes the value 2. It is $y = 2$.

Answer: 7 right and 2 up, with asymptotes $x = 7$ and $y = 2$. **Check:** the parent's (1, 1) goes to (8, 3), and $\dfrac{1}{8-7} + 2 = 3$. The parent's (−1, −1) goes to (6, 1), and $\dfrac{1}{6-7} + 2 = 1$.

Example 3. The graph of $y = x^3$ slides so that its point (1, 1) lands on (4, −2). Write the new equation and give its center.

1. The point moved from (1, 1) to (4, −2): across $h = 4 - 1 = 3$ and up $k = -2 - 1 = -3$.
2. So the rule is $y = (x-3)^3 - 3$.
3. The center (0, 0) moves the same way, to (3, −3).

Answer: $y = (x-3)^3 - 3$, center (3, −3). **Check:** at $x = 4$, $(4-3)^3 - 3 = -2$. The parent's (2, 8) should land on (5, 5), and $(5-3)^3 - 3 = 8 - 3 = 5$.

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