Writing the equation of a transformed graph

You have a graph, or a table, with the shape of a parent you know, moved and stretched. How do you find a rule that gives exactly that graph?

WORDS TO KNOW

Parent: the plain graph whose shape you see: $y = x^2$, $y = x^3$, $y = \sqrt{x}$ or $y = \dfrac{1}{x}$.

Translation: a slide with no change of shape.

Anchor: the parent's key point: the vertex, the center, the start point, or where the asymptotes cross. On the parent it is (0, 0).

Check point: a point you did not use to build the rule, used to test it.

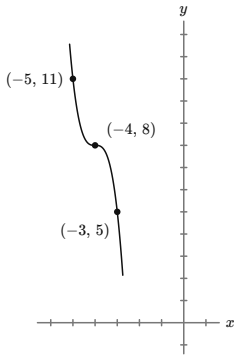
KEY IDEAS

1. The shape names the parent

Shape	Parent	Anchor
a U with one turning point	$y = x^2$	the vertex
an S that levels off for a moment	$y = x^3$	the center
half a curve, leaving a start point	$y = \sqrt{x}$	the start
two branches, each hugging two lines	$y = \dfrac{1}{x}$	where the asymptotes cross

2. The anchor gives h and k; one more point gives a

Write the rule as $y = a\,f(x - h) + k$. The anchor (0, 0) lands at (h, k), so read the anchor's coordinates. For $\dfrac{1}{x}$, the asymptotes are $x = h$ and $y = k$.



Steps: name the parent, read (h, k) from the anchor, then use another point (x, y):

$$a = \frac{y - k}{f(x - h)}$$

One step right of the anchor, $f(1) = 1$, so the height above the anchor **is** a. Below means $a < 0$.

Then test a point you did not use.

A rule needs no b: for these parents a sideways stretch is the same as an up-down one. The exception is a square root curve that runs **left** from its start. It needs $y = a\sqrt{-(x - h)} + k$.

3. Reading a table

If $g(x) = f(x - h) + k$, each entry (p, q) of f shows up in g's table as $(p + h,\ q + k)$. The outputs of g repeat the pattern of f's outputs. Match one entry that stands out, such as the smallest output, to read h and k.

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WATCH OUT

- **Mind the sign of h.** An anchor at $x = -4$ means $h = -4$, and the rule shows $x + 4$.
- **a is not rise over run.** Divide the rise from the anchor by $f(x - h)$. Two steps right on a parabola, divide by $2^2 = 4$, not by 2.
- **A curve running left needs the minus sign inside.** No outside number can turn a half curve round.
- **Always test a third point.** Two points fix the rule; only a third one checks that you read the graph and chose the parent right.

WORKED EXAMPLES

Example 1. A parabola has vertex (−2, 1) and passes through (0, −7) and (1, −17). Write its rule.

1. The U shape names $y = x^2$, so the rule is $y = a(x - h)^2 + k$.
2. The vertex is the anchor: $h = -2$ and $k = 1$. So far $y = a(x + 2)^2 + 1$.
3. Use (0, −7): $a = \dfrac{-7 - 1}{(0 + 2)^2} = \dfrac{-8}{4} = -2$.

Answer: $y = -2(x + 2)^2 + 1$. **Check:** with (1, −17), which was not used: $-2(1 + 2)^2 + 1 = -18 + 1 = -17$.

Example 2. A square root curve starts at (5, −1), runs left, and passes through (1, 5) and (−4, 8). Write its rule.

1. Half a curve from a start point names $y = \sqrt{x}$. It runs left, so it needs $b = -1$.
2. The start is the anchor: $h = 5$ and $k = -1$. So far $y = a\sqrt{-(x - 5)} - 1 = a\sqrt{5 - x} - 1$.
3. Use (1, 5): $a = \dfrac{5 - (-1)}{\sqrt{5 - 1}} = \dfrac{6}{2} = 3$.

Answer: $y = 3\sqrt{5 - x} - 1$. **Check:** with (−4, 8): $3\sqrt{5 - (-4)} - 1 = 3 \cdot 3 - 1 = 8$.

Example 3. The table of g is a translation of the table of f. Find h and k with $g(x) = f(x - h) + k$.

x	−1	0	1	2	3
$f(x)$	5	2	−2	1	4

x	3	4	5	6	7
$g(x)$	1	−2	−6	−3	0

1. The outputs of g step down 3, down 4, up 3, up 3, the same steps as f. So g is a translation of f.
2. Match the smallest outputs: f has −2 at $x = 1$, and g has −6 at $x = 5$.
3. The point (1, −2) moved to (5, −6): right 4 and down 4. So $h = 4$ and $k = -4$.

Answer: $g(x) = f(x - 4) - 4$. **Check:** $g(7) = f(3) - 4 = 4 - 4 = 0$, as in the table.

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