Algebra 2 Unit 1 quiz 1: The squaring and cubing functions, The square root and reciprocal functions, Shifting a graph up, down, left and right (answer key)

8 questions, 22 points

ANSWERS AT A GLANCE

1. $x < 0$ or $x > \frac{1}{4}$
2. (a) $0 \le x < 2.25$ (b) $x \ge 0$
3. (a) $(-2.7)^2$ (b) 2.6^3 (c) 0.3^2
4. (a) 1, 1, 1 (b) 1, 3, 5 (c) 1, 7, 19. The line keeps one slope; the parabola and the cubic get steeper as x grows, the cubic much faster.
5. (a) Domain $4 \le s \le 12$; range $16 \le A \le 144$. (b) 150: no, it is more than 144. 50: yes, with side $\sqrt{50} = 5\sqrt{2}$ cm, about 7.07 cm.
6. (a) 2, 10, −2 and −10. Her curve stays between −1 and 1 there, but the true points lie above $y = 1$ for $0 < x < 1$ and below $y = -1$ for $-1 < x < 0$. (b) From $x = 1$ toward 0 the graph climbs without bound beside the y-axis; from $x = -1$ toward 0 it falls without bound. There is no point at $x = 0$, since $\frac{1}{0}$ has no value, so the graph is two separate pieces with the asymptote $x = 0$ between them.
7. (a) Moved 4 right and 2 up; starting point (4, 2) (b) Domain $x \ge 4$, range $y \ge 2$ (c) $x = 29$
8. (a) 4 left and 2 down; $y = (x + 4)^3 - 2$ (b) The parent's (1, 1) lands on (−3, −1), which is marked, and $(-3 + 4)^3 - 2 = -1$ (c) $x = -4 + \sqrt[3]{2}$

WORKED SOLUTIONS

1. Find every $x \neq 0$ with $\frac{1}{x} < 4$. (2 points)

ANSWER

$x < 0$ or $x > \frac{1}{4}$

SOLUTION

Take the two pieces of the graph separately.

For $x > 0$, multiplying by x keeps the direction: $1 < 4x$, so $x > \frac{1}{4}$.

For $x < 0$, $\frac{1}{x}$ is negative, so it is less than 4 for every such x.

So $x < 0$ or $x > \frac{1}{4}$. (Multiplying by x without splitting would lose the whole left piece.)

SCORING

1 point for $x > \frac{1}{4}$ on the right piece; 1 point for including every $x < 0$, with the reason that $\frac{1}{x}$ is negative there.

2. (2 points)

(a) Find every x with $\sqrt{x} < 1.5$.

(b) Find every x with $\sqrt{x} > -2$.

ANSWER

(a) $0 \le x < 2.25$ (b) $x \ge 0$

SOLUTION

The domain of $\sqrt{x}$ is $x \ge 0$, so no negative x can be an answer to either part.

(a) The square root always rises, and $\sqrt{2.25} = 1.5$ since $1.5^2 = 2.25$. So for $x \ge 0$, $\sqrt{x} < 1.5$ exactly when $x < 2.25$. Together with the domain: $0 \le x < 2.25$. (Writing only $x < 2.25$ would include negative inputs, which have no square root.)

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(b) Every output of $\sqrt{x}$ is at least 0, so it is greater than −2 for every x in the domain: $x \geq 0$. (Squaring both sides to get $x > 4$ is wrong: squaring does not keep the direction when one side is negative, and $\sqrt{1} = 1 > -2$ already shows $x = 1$ works.)

SCORING

1 point for (a) $0 \leq x < 2.25$, including the left end $x = 0$ and no negative inputs; 1 point for (b) $x \geq 0$, with the reason that $\sqrt{x}$ is never negative.

3. Without working out any of the powers, decide which number is greater, and give the reason.

(a) $(-2.7)^2$ or 2.6^2 (b) $(-2.7)^3$ or 2.6^3 (c) 0.3^2 or 0.3^3 (3 points)

ANSWER

(a) $(-2.7)^2$ (b) 2.6^3 (c) 0.3^2

SOLUTION

(a) Squaring forgets the sign, so $(-2.7)^2 = 2.7^2$. The squaring graph rises for $x \geq 0$ and 2.7 > 2.6, so $(-2.7)^2 > 2.6^2$.

(b) Cubing keeps the sign: $(-2.7)^3$ is negative and 2.6^3 is positive, so 2.6^3 is greater.

(c) $0.3^3 = 0.3^2 \cdot 0.3$, and multiplying a positive number by 0.3, which is less than 1, makes it smaller. So $0.3^2 > 0.3^3$.

(The values are 7.29 > 6.76, −19.683 < 17.576 and 0.09 > 0.027.)

SCORING

1 point for each part: the greater number with a reason from the sign or the shape of the graph. A part answered only by computing both powers earns half a point.

4. On each graph, find the slope of the segment joining the points with inputs 0 and 1, the points with inputs 1 and 2, and the points with inputs 2 and 3.

(a) $y = x$ (b) $y = x^2$ (c) $y = x^3$

Then say what the slopes show about the shapes of the three graphs. (3 points)

ANSWER

(a) 1, 1, 1 (b) 1, 3, 5 (c) 1, 7, 19. The line keeps one slope; the parabola and the cubic get steeper as x grows, the cubic much faster.

SOLUTION

Each segment runs 1 unit across, so its slope is the change in output.

(a) 1 − 0 = 1, 2 − 1 = 1, 3 − 2 = 1.

(b) The points are (0, 0), (1, 1), (2, 4), (3, 9): slopes 1 − 0 = 1, 4 − 1 = 3, 9 − 4 = 5.

(c) The points are (0, 0), (1, 1), (2, 8), (3, 27): slopes 1 − 0 = 1, 8 − 1 = 7, 27 − 8 = 19.

The identity graph is a straight line, so every slope is the same. The squaring and cubing graphs are not lines: for $x \geq 0$ their slopes keep growing, so they bend upward, and the cubic climbs fastest. All three start with slope 1 from (0, 0) to (1, 1), the point they share.

SCORING

1 point for the slopes in (b); 1 point for the slopes in (c); 1 point for saying the line's slope stays the same while the other two graphs grow steeper, the cubic fastest. Part (a) alone earns no point.

5. Priya cuts square coasters with sides from 4 cm to 12 cm long. A coaster with side s cm has area $A = s^2$ square centimeters. (3 points)

(a) Give the domain and range of A in this setting.

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(b) Can a coaster have an area of 150 square centimeters? Of 50 square centimeters? For each yes, give the exact side.

ANSWER

(a) Domain $4 \le s \le 12$; range $16 \le A \le 144$. (b) 150: no, it is more than 144. 50: yes, with side $\sqrt{50} = 5\sqrt{2}$ cm, about 7.07 cm.

SOLUTION

(a) The sides run from 4 to 12, so the domain is $4 \le s \le 12$. Every input is positive, where the squaring graph rises, so the least area comes from the shortest side and the greatest from the longest: $4^2 = 16$ and $12^2 = 144$. The graph is unbroken, so the range is $16 \le A \le 144$.

(b) 150 is more than 144, outside the range: no coaster has that area. 50 is inside the range. $s^2 = 50$ gives $s = \pm\sqrt{50} = \pm 5\sqrt{2}$, but a side is positive, so $s = 5\sqrt{2} \approx 7.07$ cm, which lies between 4 and 12.

SCORING

1 point for the domain and range; 1 point for "no" for 150 with the reason; 1 point for the side $5\sqrt{2}$ cm (the negative root rejected).

6. Sarah plots $y = \dfrac{1}{x}$ at $x = -2$, -1, 1 and 2, then joins the four points with one smooth curve that passes through the origin. (3 points)

(a) Find y at $x = 0.5$, 0.1, -0.5 and -0.1. Are these points above or below Sarah's curve?

(b) Describe what the true graph does between $x = -1$ and $x = 1$, and explain why it cannot be one unbroken curve.

ANSWER

(a) 2, 10, -2 and -10. Her curve stays between -1 and 1 there, but the true points lie above $y = 1$ for $0 < x < 1$ and below $y = -1$ for $-1 < x < 0$. (b) From $x = 1$ toward 0 the graph climbs without bound beside the y-axis; from $x = -1$ toward 0 it falls without bound. There is no point at $x = 0$, since $\dfrac{1}{0}$ has no value, so the graph is two separate pieces with the asymptote $x = 0$ between them.

SOLUTION

(a) $\dfrac{1}{0.5} = 2$, $\dfrac{1}{0.1} = 10$, $\dfrac{1}{-0.5} = -2$ and $\dfrac{1}{-0.1} = -10$. Between $x = 0$ and $x = 1$ Sarah's curve runs from 0 up to 1, but the true outputs there are greater than 1, and they grow as x nears 0. Between $x = -1$ and 0 the true outputs are less than -1, not between -1 and 0.

(b) As x shrinks toward 0 through positive numbers, $\dfrac{1}{x}$ grows without bound, so the right piece shoots up beside the y-axis; through negative numbers it falls without bound, so the left piece plunges down beside it. The input 0 is not in the domain, because division by zero has no answer, so the graph has no point on the y-axis and cannot cross it. The line $x = 0$ is a vertical asymptote, and the graph is two pieces that never join. Points on one piece must not be joined to points on the other.

SCORING

1 point for the four outputs 2, 10, -2, -10 with the comparison (above 1 on the right, below -1 on the left); 1 point for the climb and fall without bound beside the y-axis; 1 point for no point at $x = 0$, so two pieces.

7. Let $g(x) = \sqrt{x - 4} + 2$. (3 points)

(a) Describe how the graph of g is moved from $y = \sqrt{x}$, and give its starting point.

(b) Give the domain and the range of g.

(c) Solve $g(x) = 7$.

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ANSWER

(a) Moved 4 right and 2 up; starting point (4, 2) (b) Domain $x \geq 4$, range $y \geq 2$ (c) $x = 29$

SOLUTION

(a) $g(x) = f(x - 4) + 2$ with $f(x) = \sqrt{x}$, so $h = 4$ and $k = 2$: the graph moves 4 right and 2 up, and the starting point (0, 0) goes to (4, 2).

(b) The square root needs $x - 4 \geq 0$, so the domain is $x \geq 4$. The parent's outputs are $y \geq 0$; adding 2 gives the range $y \geq 2$.

(c) $\sqrt{x - 4} = 7 - 2 = 5$. Square both sides: $x - 4 = 25$, so $x = 29$. Check: $\sqrt{25} + 2 = 5 + 2 = 7$.

SCORING

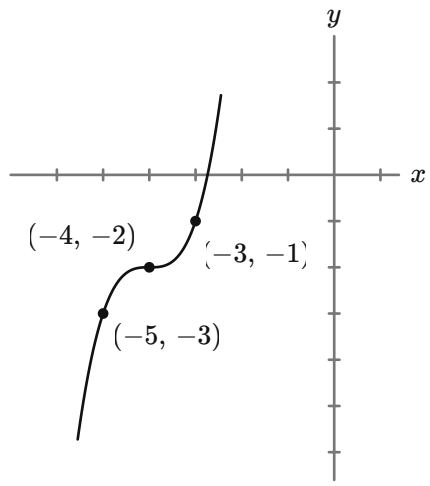
1 point: both moves with the right directions and the starting point (4, 2). 1 point: domain $x \geq 4$ and range $y \geq 2$. 1 point: $x = 29$ with the work shown.

8. The figure shows the graph of $y = x^3$ after a translation, with three of its points marked. (3 points)

(a) Name the translation and write the equation of the graph.

(b) Use another marked point to check that the shape has not changed.

(c) Find the exact x-intercept of the graph.



ANSWER

(a) 4 left and 2 down; $y = (x + 4)^3 - 2$ (b) The parent's (1, 1) lands on (−3, −1), which is marked, and $(-3 + 4)^3 - 2 = -1$ (c) $x = -4 + \sqrt[3]{2}$

SOLUTION

(a) The anchor of $y = x^3$ is its center (0, 0), where the curve flattens. Here the curve flattens at (−4, −2), so $h = -4$ and $k = -2$: the graph moved 4 left and 2 down, and

$$y = (x - (-4))^3 + (-2) = (x + 4)^3 - 2$$

(b) The parent's (1, 1) should land on (1 − 4, 1 − 2) = (−3, −1), which is marked, and the formula agrees: $(-3 + 4)^3 - 2 = 1 - 2 = -1$. The parent's (−1, −1) should land on (−5, −3), also marked: $(-1)^3 - 2 = -3$.

(c) Set $y = 0$: $(x + 4)^3 = 2$, so $x + 4 = \sqrt[3]{2}$ and $x = -4 + \sqrt[3]{2}$, about −2.74.

SCORING

1 point: 4 left and 2 down with the equation $y = (x + 4)^3 - 2$. 1 point: a second marked point carried from the parent and tested in the formula. 1 point: $x = -4 + \sqrt[3]{2}$.

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