Algebra 2 Unit 1 quiz 2: Reflecting a graph across the axes, Stretches and compressions, Combining transformations and the general form (answer key)

8 questions, 22 points

ANSWERS AT A GLANCE

1. $g(4) = 11$ and $h(4) = 29$
2. $(-2, 14)$, the highest point of g
3. (a) A horizontal stretch by a factor of 2. (b) f: x-intercepts -8 and 0, vertex $(-4, -16)$. g: x-intercepts -16 and 0, vertex $(-8, -16)$. (c) $g(x) = \frac{1}{4}x^2 + 4x$; range $y \geq -16$.
4. (a) $a = -\frac{1}{2}$, $b = -4$, $h = 2$, $k = 9$. (b) Horizontally: compress by a factor of $\frac{1}{4}$ toward the y-axis and reflect across the y-axis, then shift right 2. Vertically: compress by a factor of $\frac{1}{2}$ and reflect across the x-axis, then shift up 9. (c) $(0, 0) \mapsto (2, 9)$ and $(1, 1) \mapsto \left(\frac{7}{4}, \frac{17}{2}\right)$.
5. $g(x) = f(-x)$; $g(-2) = 5$ and $g(1) = -2$.
6. (a) The y-axis. (b) $g(x) = f(-x) = \frac{1}{-x - 2}$, which is $-\frac{1}{x + 2}$; domain: every $x \neq -2$.
7. (a) A vertical compression by a factor of $\frac{1}{12}$. (b) $F(t)$: -0.5, 0, 1.5, 2.5, 0.75. (c) 27 inches above normal.
8. (a) $a = \frac{1}{2}$: a vertical compression by a factor of $\frac{1}{2}$. (b) The x-intercepts $(-2, 0)$ and $(2, 0)$. (c) f: $-2 \leq y \leq 4$; g: $-1 \leq y \leq 2$.

WORKED SOLUTIONS

1. Let $f(x) = x^2 - 5x - 7$, $g(x) = -f(x)$ and $h(x) = f(-x)$. Find $g(4)$ and $h(4)$. (2 points)

> **ANSWER**
> $g(4) = 11$ and $h(4) = 29$
>
> **SOLUTION**
> g takes the output of f and changes its sign: $f(4) = 16 - 20 - 7 = -11$, so $g(4) = -f(4) = 11$.
>
> h hands f the opposite of its input: $h(4) = f(-4) = (-4)^2 + (-5)(-4) - 7 = 16 + 20 - 7 = 29$.
>
> **SCORING**
> 1 point for $g(4) = 11$; 1 point for $h(4) = 29$.

2. The graph of a function f has its highest point at $(-9, 3)$. Let $g(x) = 4\,f(2x - 5) + 2$. Find the point of g that comes from it, and say whether it is the highest or the lowest point of g. (2 points)

> **ANSWER**
> $(-2, 14)$, the highest point of g
>
> **SOLUTION**
> The point of g comes from the input x at which the inside equals -9: $2x - 5 = -9$, so $2x = -4$ and $x = -2$. There $g(-2) = 4\,f(-9) + 2 = 4 \cdot 3 + 2 = 14$. So $(-9, 3)$ goes to $(-2, 14)$.
>
> Since a is positive, the stretch keeps the heights in the same order, so the highest point of f becomes the highest point of g. The shifts and the horizontal compression move points without changing which one is highest or lowest.
>
> **SCORING**
> 1 point for the point $(-2, 14)$; 1 point for "highest" with the sign of a as the reason.

3. Let $f(x) = x^2 + 8x$ and $g(x) = f\left(\frac{1}{2}x\right)$. (3 points)

(a) Name the move from the graph of f to the graph of g, with its factor.

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(b) Find the x-intercepts and the vertex of the graph of f. Use them to give the x-intercepts and the vertex of the graph of g.

(c) Write $g(x)$ as a polynomial in x, and give the range of g.

ANSWER

(a) A horizontal stretch by a factor of 2. (b) f: x-intercepts -8 and 0, vertex $(-4, -16)$. g: x-intercepts -16 and 0, vertex $(-8, -16)$. (c) $g(x) = \frac{1}{4}x^2 + 4x$; range $y \geq -16$.

SOLUTION

(a) The input is multiplied by $b = \frac{1}{2}$, so every point moves to $\frac{1}{b} = 2$ times its distance from the y-axis: $(p, q) \mapsto (2p, q)$. The graph of g is the graph of f stretched horizontally by a factor of 2.

(b) $f(x) = x(x + 8) = 0$ gives $x = 0$ and $x = -8$. Completing the square, $f(x) = (x + 4)^2 - 16$, so the vertex is $(-4, -16)$. Doubling each input: the x-intercepts of g are 0 and -16, and its vertex is $(-8, -16)$.

(c) $g(x) = \left(\frac{1}{2}x\right)^2 + 8 \cdot \frac{1}{2}x = \frac{1}{4}x^2 + 4x$. Check: $g(-16) = 64 - 64 = 0$ and $g(-8) = 16 - 32 = -16$. A horizontal move does not change the outputs, so the range is that of f, $y \geq -16$.

SCORING

1 point: horizontal stretch with factor 2 (not $\frac{1}{2}$). 1 point: x-intercepts -16 and 0 and vertex $(-8, -16)$ of g. 1 point: $g(x) = \frac{1}{4}x^2 + 4x$ and range $y \geq -16$.

4. The rule $y = -\frac{1}{2}(8 - 4x)^3 + 9$ is built from $y = x^3$. (3 points)

(a) Give a, b, h and k.

(b) Describe the moves in order.

(c) Where do the parent's center $(0, 0)$ and its point $(1, 1)$ go?

ANSWER

(a) $a = -\frac{1}{2}$, $b = -4$, $h = 2$, $k = 9$. (b) Horizontally: compress by a factor of $\frac{1}{4}$ toward the y-axis and reflect across the y-axis, then shift right 2. Vertically: compress by a factor of $\frac{1}{2}$ and reflect across the x-axis, then shift up 9. (c) $(0, 0) \mapsto (2, 9)$ and $(1, 1) \mapsto \left(\frac{7}{4}, \frac{17}{2}\right)$.

SOLUTION

(a) Factor the inside: $8 - 4x = -4(x - 2)$. So $y = -\frac{1}{2}\big(-4(x - 2)\big)^3 + 9$, with $a = -\frac{1}{2}$, $b = -4$, $h = 2$, $k = 9$.

(b) The horizontal pair: $b = -4$ compresses by $\frac{1}{4}$ toward the y-axis and reflects across the y-axis, and then $h = 2$ shifts right 2. The vertical pair: $a = -\frac{1}{2}$ compresses by $\frac{1}{2}$ and reflects across the x-axis, and then $k = 9$ shifts up 9. Within each pair the stretch or flip comes before the shift.

(c) A point goes to $\left(\frac{p}{-4} + 2,\ -\frac{1}{2}q + 9\right)$. So $(0, 0) \mapsto (2, 9)$, and $(1, 1) \mapsto \left(-\frac{1}{4} + 2,\ -\frac{1}{2} + 9\right) = \left(\frac{7}{4}, \frac{17}{2}\right)$. Check: $8 - 4 \cdot \frac{7}{4} = 1$, and $-\frac{1}{2} \cdot 1^3 + 9 = \frac{17}{2}$.

SCORING

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1 point for all four of a, b, h, k; 1 point for the moves with each stretch or flip before its shift; 1 point for both image points.

5. Here are a table for a function f and a table for a function g.

x	−3	−1	0	2	4
$f(x)$	6	−2	1	5	−4

x	−4	−2	0	1	3
$g(x)$	−4	?	1	?	6

The function g is either $-f(x)$ or $f(-x)$, and every entry in its table was worked out from an entry in the table of f. Decide which it is, giving a reason, and fill in the two missing entries. (3 points)

ANSWER

$g(x) = f(-x)$; $g(-2) = 5$ and $g(1) = -2$.

SOLUTION

At $x = 0$ the two choices differ: $-f(0) = -1$ but $f(-0) = f(0) = 1$, and $g(0) = 1$. Also, $-f(-4)$ could not be worked out, because $f(-4)$ is not in the table, but $f(-(-4)) = f(4) = -4$ is, and it matches $g(-4) = -4$. Likewise $g(3) = f(-3) = 6$. So $g(x) = f(-x)$.

Then $g(-2) = f(2) = 5$ and $g(1) = f(-1) = -2$.

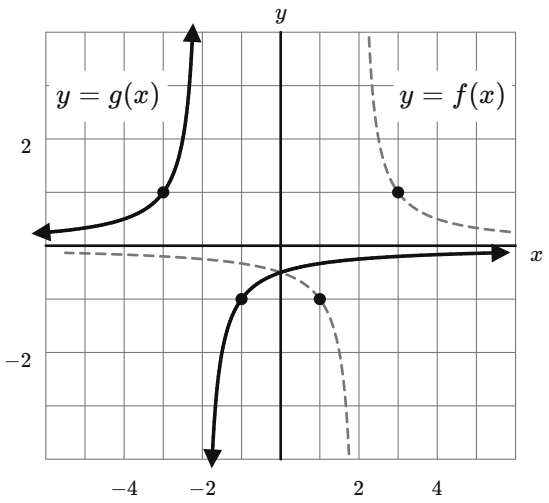
SCORING

1 point for $g(x) = f(-x)$; 1 point for a correct reason, such as $g(0) = 1 = f(0)$ while $-f(0) = -1$, or matching $g(-4)$ with $f(4)$; 1 point for both missing entries, 5 and −2.

6. The dashed graph is $f(x) = \dfrac{1}{x-2}$. The solid graph g is the graph of f reflected across one of the axes. (3 points)

(a) Which axis? Give a reason from matching points.

(b) Write a rule for $g(x)$ and give the domain of g.



ANSWER

(a) The y-axis. (b) $g(x) = f(-x) = \dfrac{1}{-x-2}$, which is $-\dfrac{1}{x+2}$; domain: every $x \neq -2$.

SOLUTION

(a) The marked point (3, 1) of f has the partner (−3, 1) on g, and (1, −1) has the partner (−1, −1): the same output at the opposite input. That is $(a, b) \mapsto (-a, b)$, a reflection across the y-axis. (A reflection across the x-axis would keep the gap at $x = 2$, and g's gap is at $x = -2$.)

(b) $g(x) = f(-x) = \dfrac{1}{-x-2}$, which can also be written $-\dfrac{1}{x+2}$. Check: $g(-3) = \dfrac{1}{3-2} = 1$. The denominator is 0 when $-x - 2 = 0$, at $x = -2$, so the domain is every $x \neq -2$: the gap of f at $x = 2$ has moved across the y-axis.

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SCORING

1 point for the y-axis with a reason from matching points such as (3, 1) and (−3, 1); 1 point for $g(x) = \dfrac{1}{-x-2}$ or an equal form; 1 point for the domain $x \neq -2$.

7. A river gauge at a town bridge reports the water level $L(t)$, in inches above (positive) or below (negative) normal, t hours after midnight. The town's flood app shows the same readings in feet, $F(t) = \dfrac{1}{12}L(t)$.

(3 points)

t	0	3	6	9	12
$L(t)$	−6	0	18	30	9

(a) Name the move from the graph of L to the graph of F.

(b) Make a table for F, in decimals.

(c) The app sounds an alert when the level reaches 2.25 feet above normal. What gauge reading, in inches, is that?

ANSWER

(a) A vertical compression by a factor of $\dfrac{1}{12}$. (b) $F(t)$: −0.5, 0, 1.5, 2.5, 0.75. (c) 27 inches above normal.

SOLUTION

(a) Every output of L is multiplied by $\dfrac{1}{12}$, which is between 0 and 1, and the times stay, so the graph of F is the graph of L compressed vertically by a factor of $\dfrac{1}{12}$.

(b) Divide each reading by 12:

t	0	3	6	9	12
$F(t)$	−0.5	0	1.5	2.5	0.75

The reading 0 at $t = 3$ is the only one that does not change: it is on the t-axis.

(c) Undo the compression: $L = 12F = 12 \cdot 2.25 = 27$. The alert matches a gauge reading of 27 inches above normal.

SCORING

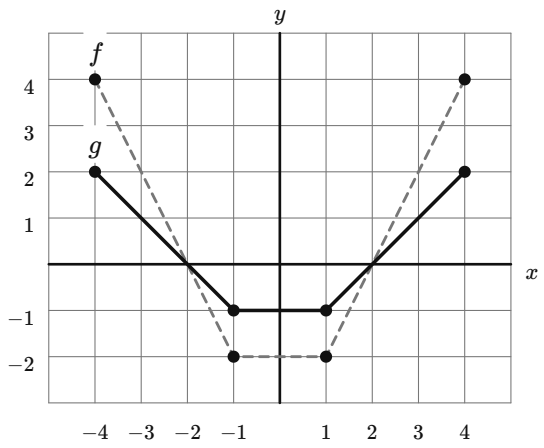
1 point: vertical compression with factor $\dfrac{1}{12}$. 1 point: all five values of F correct. 1 point: 27 inches.

8. In the figure, the solid graph g is the dashed graph f after a vertical stretch or compression, $g(x) = a\,f(x)$. (3 points)

(a) Find a, check it with a second pair of points, and name the move.

(b) Which points of the graph of f did not move?

(c) Give the range of f and the range of g.



ANSWER

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(a) $a = \frac{1}{2}$: a vertical compression by a factor of $\frac{1}{2}$. (b) The x-intercepts $(-2, 0)$ and $(2, 0)$. (c) f: $-2 \le y \le 4$; g: $-1 \le y \le 2$.

SOLUTION

(a) Compare outputs at the same input. At $x = -4$, $f(-4) = 4$ and $g(-4) = 2$, so $a = \frac{2}{4} = \frac{1}{2}$. Check at $x = 1$: $f(1) = -2$ and $g(1) = -1$, and $\frac{1}{2} \cdot (-2) = -1$. So g is f compressed vertically by a factor of $\frac{1}{2}$.

(b) Only the points with $f(x) = 0$ stay, since $\frac{1}{2} \cdot 0 = 0$. The segment from $(-4, 4)$ to $(-1, -2)$ falls 6 over 3 units, so it reaches 0 two units along, at $(-2, 0)$. The segment from $(1, -2)$ to $(4, 4)$ rises 6 over 3 units and reaches 0 at $(2, 0)$. The middle segment stays at height -2. The solid graph passes through the same two points.

(c) The lowest outputs of f are -2 (the flat middle segment) and the highest is 4 (both ends), so f has range $-2 \le y \le 4$. Halving every output gives g the range $-1 \le y \le 2$. (Both have domain $-4 \le x \le 4$.)

SCORING

1 point: $a = \frac{1}{2}$ from one pair, confirmed with a second, named as a vertical compression. 1 point: both fixed points and no others. 1 point: both ranges.

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