Algebra 2 Unit 1 quiz 3: Graphing by transforming a parent graph, Finding the rule of a transformed graph or table, Transforming the absolute value graph (answer key)

7 questions, 23 points

ANSWERS AT A GLANCE

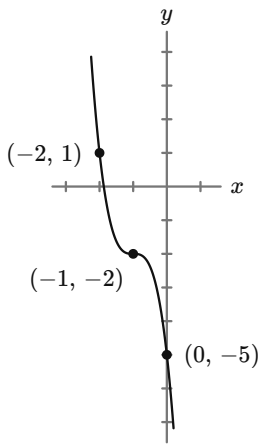
1. (a) $a = -3$; $y = -3(x+1)^3 - 2$ (b) $-3(1)^3 - 2 = -5$
2. $a = -4$, $k = 8$; $y = -4|x+2| + 8$; range $y \le 8$.
3. (a) Endpoint (−1, −8); the graph runs left and up. (b) Domain $(-\infty, -1]$, range $[-8, \infty)$. (c) (−5, 0)
4. Images: $(0,0) \mapsto (-3,-8)$, $(2,4) \mapsto (-2,-6)$, $(-2,4) \mapsto (-4,-6)$, $(4,16) \mapsto (-1,0)$, $(-4,16) \mapsto (-5,0)$. The sketch is an upward parabola through these points with axis $x = -3$. Range $[-8, \infty)$; x-intercepts (−5, 0) and (−1, 0), y-intercept (0, 10).
5. (a) Asymptotes $x = 2$ and $y = -2$; $(1,1) \mapsto \left(\frac{3}{2}, 1\right)$, $\left(2, \frac{1}{2}\right) \mapsto \left(1, -\frac{1}{2}\right)$, $(-1,-1) \mapsto \left(\frac{5}{2}, -5\right)$, $\left(-2, -\frac{1}{2}\right) \mapsto \left(3, -\frac{7}{2}\right)$. (b) One branch up and to the left of the center (2, −2), the other down and to the right. (c) Domain $(-\infty, 2) \cup (2, \infty)$, range $(-\infty, -2) \cup (-2, \infty)$; x-intercept $\left(\frac{5}{4}, 0\right)$.
6. (a) $y = \frac{4}{x-2} - 1$; at (−2, −2): $\frac{4}{-4} - 1 = -2$ (b) x-intercept (6, 0), y-intercept (0, −3)
7. (a) The heights at $x = 2$ and $x = 3$ are equal, so the vertex is halfway between them: $\left(\frac{5}{2}, -1\right)$. (b) $y = 2|x - \frac{5}{2}| - 1$, which is also $y = |2x - 5| - 1$. (c) $y \ge -1$

WORKED SOLUTIONS

1. The figure shows a transformed graph of $y = x^3$, with its center marked. (2 points)

(a) Use the marked point (−2, 1) to find a, and write the rule.

(b) Check your rule at (0, −5).



ANSWER

(a) $a = -3$; $y = -3(x+1)^3 - 2$ (b) $-3(1)^3 - 2 = -5$

SOLUTION

(a) The center (−1, −2) gives $h = -1$ and $k = -2$: $y = a(x+1)^3 - 2$. At (−2, 1):

$$a = \frac{1 - (-2)}{(-2+1)^3} = \frac{3}{-1} = -3$$

The point is 3 **above** the center, yet a is negative: it lies one step to the left, where x^3 gives −1. The rule is $y = -3(x+1)^3 - 2$, which falls from left to right, as the graph does.

(b) At $x = 0$: $-3(0+1)^3 - 2 = -3 - 2 = -5$, the marked point.

2. A V with a rule of the form $y = a|x - h| + k$ has axis of symmetry $x = -2$ and passes through (1, −4) and (−4, 0). Find a and k, write the rule, and give the range. (3 points)

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ANSWER

$a = -4$, $k = 8$; $y = -4|x + 2| + 8$; range $y \le 8$.

SOLUTION

The axis gives $h = -2$, so $y = a|x + 2| + k$. The two points give two equations:

$(1, -4)$: $a|1 + 2| + k = -4$, so $3a + k = -4$.

$(-4, 0)$: $a|-4 + 2| + k = 0$, so $2a + k = 0$.

Subtracting, $a = -4$, and then $k = -2a = 8$. The rule is $y = -4|x + 2| + 8$, with vertex $(-2, 8)$. Since $a < 0$ the vertex is the highest point and the range is $y \le 8$.

Check: $-4 \cdot 3 + 8 = -4$ and $-4 \cdot 2 + 8 = 0$.

SCORING

1 point: both equations $3a + k = -4$ and $2a + k = 0$ (or equivalent). 1 point: $a = -4$ and $k = 8$, with the rule $y = -4|x + 2| + 8$. 1 point: range $y \le 8$.

3. Let $g(x) = 2\sqrt{-4 - 4x} - 8$. (4 points)

(a) Give the endpoint of the graph of g, and say which way the graph runs from it.

(b) Give the domain and range of g in interval notation.

(c) Find the x-intercept of the graph.

ANSWER

(a) Endpoint $(-1, -8)$; the graph runs left and up. (b) Domain $(-\infty, -1]$, range $[-8, \infty)$. (c) $(-5, 0)$

SOLUTION

Factor the inside: $-4 - 4x = -4(x + 1)$. So $g(x) = 2\sqrt{-4(x + 1)} - 8$ with $a = 2$, $b = -4$, $h = -1$ and $k = -8$.

(a) The parent's endpoint $(0, 0)$ goes to $(h, k) = (-1, -8)$. Since $b < 0$ the graph turns left for right and runs to the left; the sign of $a = 2$ makes it run up.

(b) Domain: $-4 - 4x \ge 0$ exactly when $x \le -1$, so the domain is $(-\infty, -1]$; the negative b swapped the ends of $[0, \infty)$. Range: each height $q \ge 0$ of the parent becomes $2q - 8$, so the range is $[-8, \infty)$.

(c) Set $g(x) = 0$: $2\sqrt{-4 - 4x} = 8$, so $\sqrt{-4 - 4x} = 4$. Squaring, $-4 - 4x = 16$, so $x = -5$. Check: $2\sqrt{16} - 8 = 0$. The intercept $(-5, 0)$ lies to the left of the endpoint, as it must.

SCORING

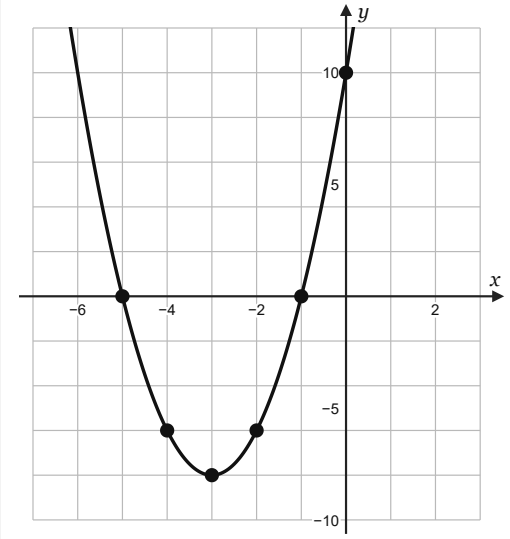
1 point for the endpoint $(-1, -8)$ with the direction left and up; 1 point for the domain; 1 point for the range; 1 point for the x-intercept $(-5, 0)$.

4. Graph $y = \frac{1}{2}(2x + 6)^2 - 8$ by moving $y = x^2$. Carry the vertex $(0, 0)$ and the parent points $(\pm 2, 4)$ and $(\pm 4, 16)$, sketch the graph, and give its range and every point where it meets an axis. (4 points)

ANSWER

Images: $(0, 0) \mapsto (-3, -8)$, $(2, 4) \mapsto (-2, -6)$, $(-2, 4) \mapsto (-4, -6)$, $(4, 16) \mapsto (-1, 0)$, $(-4, 16) \mapsto (-5, 0)$. The sketch is an upward parabola through these points with axis $x = -3$. Range $[-8, \infty)$; x-intercepts $(-5, 0)$ and $(-1, 0)$, y-intercept $(0, 10)$.

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SOLUTION

Factor the inside: $2x + 6 = 2(x + 3)$, so $a = \dfrac{1}{2}$, $b = 2$, $h = -3$, $k = -8$. Each point (p, q) goes to $\left(\dfrac{p}{2} - 3,\ \dfrac{q}{2} - 8\right)$:

Parent point	x: $\frac{p}{2} - 3$	y: $\frac{q}{2} - 8$	**Image**
$(-4, 16)$	-5	0	$(-5, 0)$
$(-2, 4)$	-4	-6	$(-4, -6)$
$(0, 0)$	-3	-8	$(-3, -8)$
$(2, 4)$	-2	-6	$(-2, -6)$
$(4, 16)$	-1	0	$(-1, 0)$

The vertex is $(-3, -8)$ and $a > 0$, so the parabola opens upward with axis $x = -3$. Range $[-8, \infty)$.

The images with height 0 are the x-intercepts, $(-5, 0)$ and $(-1, 0)$. For the y-intercept put $x = 0$: $\dfrac{1}{2}(6)^2 - 8 = 18 - 8 = 10$, so $(0, 10)$.

SCORING

1 point for the factored form or the vertex $(-3, -8)$; 1 point for all five image points; 1 point for an upward parabola through them; 1 point for the range $[-8, \infty)$ and the three intercepts.

5. Graph $y = \dfrac{3}{4 - 2x} - 2$ by moving $y = \dfrac{1}{x}$. (4 points)

(a) Give the asymptotes, and carry the parent points $(1, 1)$, $\left(2, \dfrac{1}{2}\right)$, $(-1, -1)$ and $\left(-2, -\dfrac{1}{2}\right)$.

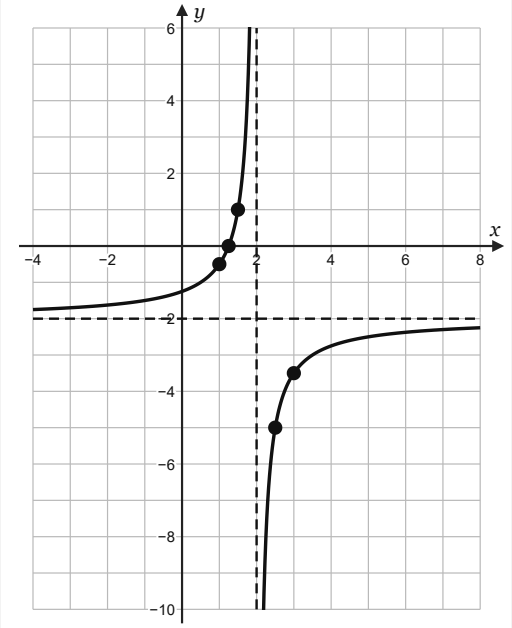
(b) Sketch the graph, and say where its branches sit around the center.

(c) Give the domain and range, and the x-intercept.

ANSWER

(a) Asymptotes $x = 2$ and $y = -2$; $(1, 1) \mapsto \left(\dfrac{3}{2}, 1\right)$, $\left(2, \dfrac{1}{2}\right) \mapsto \left(1, -\dfrac{1}{2}\right)$, $(-1, -1) \mapsto \left(\dfrac{5}{2}, -5\right)$, $\left(-2, -\dfrac{1}{2}\right) \mapsto \left(3, -\dfrac{7}{2}\right)$. (b) One branch up and to the left of the center $(2, -2)$, the other down and to the right. (c) Domain $(-\infty, 2) \cup (2, \infty)$, range $(-\infty, -2) \cup (-2, \infty)$; x-intercept $\left(\dfrac{5}{4}, 0\right)$.

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SOLUTION

(a) $4-2x=-2(x-2)$, so $\dfrac{3}{4-2x}=3\cdot\dfrac{1}{-2(x-2)}$, with $a=3$, $b=-2$, $h=2$, $k=-2$. The asymptotes move to $x=2$ and $y=-2$. Each point goes to $\left(\dfrac{p}{-2}+2,\ 3q-2\right)$:

$$(1,1)\mapsto\left(\tfrac{3}{2},1\right)\qquad(-1,-1)\mapsto\left(\tfrac{5}{2},-5\right)$$
$$\left(2,\tfrac{1}{2}\right)\mapsto\left(1,-\tfrac{1}{2}\right)\quad\left(-2,-\tfrac{1}{2}\right)\mapsto\left(3,-\tfrac{7}{2}\right)$$

Check: $\dfrac{3}{4-3}-2=1$.

(b) The negative b turns the graph left for right, so the parent's up-right branch now sits up and to the left of the center $(2,-2)$, through $\left(1,-\dfrac{1}{2}\right)$ and $\left(\dfrac{3}{2},1\right)$, and the other branch sits down and to the right, through $\left(\dfrac{5}{2},-5\right)$ and $\left(3,-\dfrac{7}{2}\right)$. Each branch bends toward both dashed asymptotes without reaching them.

(c) The gaps at 0 move with the asymptotes: domain $(-\infty,2)\cup(2,\infty)$, range $(-\infty,-2)\cup(-2,\infty)$. For the x-intercept, $\dfrac{3}{4-2x}=2$ gives $4-2x=\dfrac{3}{2}$, so $x=\dfrac{5}{4}$: the point $\left(\dfrac{5}{4},0\right)$, on the upper-left branch.

SCORING

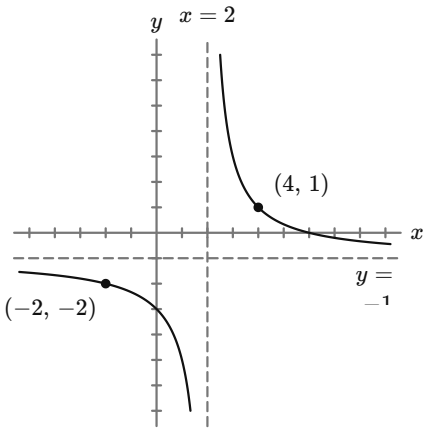
1 point for both asymptotes; 1 point for the four image points; 1 point for a sketch with the branches up-left and down-right of $(2,-2)$; 1 point for the domain, range and x-intercept $\left(\dfrac{5}{4},0\right)$.

6. The figure shows a transformed graph of $y=\dfrac{1}{x}$ with its asymptotes dashed. (3 points)

(a) Write its rule, and check it with the marked point you did not use.

(b) Use your rule to find the x-intercept and the y-intercept of the graph.

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ANSWER

(a) $y = \dfrac{4}{x-2} - 1$; at (−2, −2): $\dfrac{4}{-4} - 1 = -2$ (b) x-intercept (6, 0), y-intercept (0, −3)

SOLUTION

(a) The asymptotes $x = 2$ and $y = -1$ give $h = 2$ and $k = -1$: $y = \dfrac{a}{x-2} - 1$. At (4, 1), two steps right of $x = 2$:

$$a = \frac{1 - (-1)}{\frac{1}{4-2}} = 2 \cdot 2 = 4$$

So $y = \dfrac{4}{x-2} - 1$. Check with (−2, −2): $\dfrac{4}{-4} - 1 = -1 - 1 = -2$.

(b) Set $y = 0$: $\dfrac{4}{x-2} = 1$, so $x - 2 = 4$ and $x = 6$: the point (6, 0). At $x = 0$: $\dfrac{4}{-2} - 1 = -3$: the point (0, −3).

SCORING

1 point: $h = 2$ and $k = -1$ read from the asymptotes. 1 point: $a = 4$ and the rule, checked at (−2, −2). 1 point: both intercepts, (6, 0) and (0, −3).

7. The table comes from a rule of the form $y = a|x - h| + k$. (3 points)

x	0	1	2	3	4	5
y	4	2	0	0	2	4

(a) Explain why the vertex is not one of the table's points, and find it.

(b) Write the rule.

(c) Give the range.

ANSWER

(a) The heights at $x = 2$ and $x = 3$ are equal, so the vertex is halfway between them: $\left(\dfrac{5}{2}, -1\right)$. (b) $y = 2|x - \dfrac{5}{2}| - 1$, which is also $y = |2x - 5| - 1$. (c) $y \geq -1$

SOLUTION

(a) The heights fall by 2 per step from $x = 0$ to $x = 2$ and rise by 2 per step from $x = 3$ to $x = 5$. Two inputs with the same height sit at equal distances on either side of the axis, so the axis is $x = \dfrac{2+3}{2} = \dfrac{5}{2}$, between two table entries. The right arm has slope 2 and passes through (3, 0), so half a step back, at $x =$

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$\frac{5}{2}$, its height is $0 - 2 \cdot \frac{1}{2} = -1$. The vertex is $\left(\frac{5}{2}, -1\right)$, and the lowest value in the table, 0, is not the lowest value of the rule.

(b) $a = 2$ (the right arm's slope), $h = \frac{5}{2}$, $k = -1$: $y = 2|x - \frac{5}{2}| - 1$. Since $2|x - \frac{5}{2}| = |2x - 5|$, this is also $y = |2x - 5| - 1$. Check at $x = 0$: $|-5| - 1 = 4$.

(c) $a > 0$, so the vertex is the lowest point: $y \geq -1$.

SCORING

1 point: (a) explains that equal heights at $x = 2$ and $x = 3$ put the axis at $x = \frac{5}{2}$, and finds the vertex $\left(\frac{5}{2}, -1\right)$. 1 point: (b) a correct rule in either form. 1 point: (c) $y \geq -1$ (follow through from the vertex found).

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