Algebra 2 Unit 1 quiz 4: Symmetry about the vertical axis and the origin, Even and odd functions (answer key)

7 questions, 20 points

ANSWERS AT A GLANCE

1. $k = 2$; the points are $(10, 4)$ and $(-10, 4)$.
2. (a) $(-2, -4)$, $(-5, -1)$, $(-7, -3)$; range $-4 \le y \le 4$.
   (b) $x = 1$, $x = 4$ and $x = 6$.
   (c) $x = -5$ and $x = -\frac{1}{2}$.
3. (a) $f(-x) = -4x^3 + 7x = -f(x)$, so f is odd. (b) $g(-1) = 2$ and $-g(1) = 4$. (c) Neither.
4. (a) odd (b) even (c) neither
5. (a) Symmetric about the x-axis only. (b) $(-2, -1)$; it lies on the same vertical line as $(-2, 1)$.
6. (a) The entry at $x = 0$: it should be $f(0) = 0$.
   (b) No: three pairs of opposite inputs disagree, and one corrected entry can fix only one pair.
7. (a) The partner of $(1, -2)$ would be $(-1, -2)$, but $g(-1) = 6$.
   (b) $h = 2$:

x	−3	−2	−1	0	1	2	3
$g(x+2)$	6	1	−2	−3	−2	1	6

   (c) No: $g(1) + c$ and $g(-1) + c$ still differ by 8 for every c.

WORKED SOLUTIONS

1. A graph is symmetric about the vertical axis. The points $(k + 8, 4)$ and $(5k - 20, 4)$ are on it, and each is the other's mirror partner (they are two different points). Find k and give both points. (2 points)

ANSWER

$k = 2$; the points are $(10, 4)$ and $(-10, 4)$.

SOLUTION

Mirror partners have the same height and opposite inputs, so $k + 8 = -(5k - 20) = -5k + 20$. Then $6k = 12$ and $k = 2$.

The points are $(2 + 8, 4) = (10, 4)$ and $(5 \cdot (2) - 20, 4) = (-10, 4)$: the same height, at opposite inputs.

SCORING

1 point for setting the inputs as opposites, $k + 8 = -(5k - 20)$; 1 point for $k = 2$ and both points $(10, 4)$ and $(-10, 4)$.

2. A function f has domain $-7 \le x \le 7$, and its graph is symmetric about the origin. For $0 \le x \le 7$ the graph is made of line segments joining $(0, 0)$, $(2, 4)$, $(5, 1)$ and $(7, 3)$ in that order. (3 points)

(a) Give the corners of the graph for $-7 \le x \le 0$, and the range of f.

(b) Find every x with $f(x) = 2$.

(c) Find every x with $f(x) = -1$.

ANSWER

(a) $(-2, -4)$, $(-5, -1)$, $(-7, -3)$; range $-4 \le y \le 4$.

(b) $x = 1$, $x = 4$ and $x = 6$.

(c) $x = -5$ and $x = -\frac{1}{2}$.

SOLUTION

(a) Each (a, b) has the partner $(-a, -b)$, so the left part joins $(0, 0)$, $(-2, -4)$, $(-5, -1)$ and $(-7, -3)$. For $x \ge 0$ the heights run from 0 to 4; the half turn sends them to -4 to 0. Together the range is $-4 \le y \le 4$.

(b) For $x \ge 0$: on the first segment $y = 2x$, so $y = 2$ at $x = 1$; on the second, from $(2, 4)$ to $(5, 1)$, $y = 6 - x$, so $x = 4$; on the third, from $(5, 1)$ to $(7, 3)$, $y = x - 4$, so $x = 6$. For $x < 0$, a height of 2 would be the partner of a height of -2 for $x > 0$, and the right part is never below 0. So $f(x) = 2$ only at $x = 1$, 4 and 6.

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(c) For $x \geq 0$ the graph is never at -1. For $x < 0$, $f(x) = -1$ exactly where the partner input $-x$ has height 1. On the right, height 1 occurs on the first segment where $2x = 1$, at $x = \frac{1}{2}$, and at the corner $(5, 1)$. So $f(x) = -1$ at $x = -5$ and $x = -\frac{1}{2}$.

SCORING

1 point for the three corners and the range $-4 \leq y \leq 4$; 1 point for $x = 1$, 4, 6 and no negative solutions; 1 point for $x = -5$ and $x = -\frac{1}{2}$ (found from height 1 on the right).

3. Let $f(x) = 4x^3 - 7x$ and $g(x) = 4x^3 - 7x - 1$. (3 points)

(a) Simplify $f(-x)$ and decide whether f is even, odd or neither.

(b) Find $g(-1)$ and $-g(1)$.

(c) Is g even, odd or neither? Explain.

ANSWER

(a) $f(-x) = -4x^3 + 7x = -f(x)$, so f is odd. (b) $g(-1) = 2$ and $-g(1) = 4$. (c) Neither.

SOLUTION

(a) Both powers are odd, so each term changes sign: $f(-x) = 4(-x)^3 - 7(-x) = -4x^3 + 7x$, which is $-f(x)$. f is odd.

(b) $g(x) = f(x) - 1$, so $g(-1) = f(-1) - 1 = 2$. Also $g(1) = -4$, so $-g(1) = 4$.

(c) $g(-1) \neq -g(1)$, so g is not odd: the constant -1 keeps its sign when x becomes $-x$, so it breaks the oddness of f. And $g(-1) = 2 \neq -4 = g(1)$, so g is not even. g is neither.

SCORING

1 point for $f(-x) = -4x^3 + 7x$ with the conclusion that f is odd. 1 point for both values, 2 and 4. 1 point for "neither" with a reason for each half: the values in (b) rule out odd, and comparing with $g(1) = -4$ (or simplifying $g(-x)$) rules out even.

4. Decide whether each function is even, odd or neither. Show the work that decides it.

(a) $f(x) = \dfrac{x^3}{|x| + 1}$ (b) $g(x) = \sqrt{x^4 + 1} - 2$ (c) $h(x) = \sqrt{x^2 + 6x + 9}$ (3 points)

ANSWER

(a) odd (b) even (c) neither

SOLUTION

(a) $|x| + 1$ is never 0, so the domain is every real number. Since $|-x| = |x|$, $f(-x) = \dfrac{(-x)^3}{|x| + 1} = -\dfrac{x^3}{|x| + 1} = -f(x)$. Odd: an odd function divided by an even one.

(b) The domain is every real number. $g(-x) = \sqrt{(-x)^4 + 1} - 2 = \sqrt{x^4 + 1} - 2 = g(x)$. Even: the constant -2 is an even piece and does not spoil evenness.

(c) $x^2 + 6x + 9 = (x + 3)^2 \geq 0$, so the domain is every real number, and $h(-x) = \sqrt{x^2 - 6x + 9}$, which is not the same rule: only the $6x$ term changed sign. One input proves it: $h(3) = \sqrt{36} = 6$ and $h(-3) = \sqrt{0} = 0$. Since $0 \neq 6$ and $0 \neq -6$, h is neither. (In fact $h(x) = |x + 3|$, a V moved left 3.)

SCORING

1 point for each part: the right conclusion with the work that supports it, $f(-x) = -f(x)$ for (a), $g(-x) = g(x)$ for (b), and for (c) an input such as $x = 3$ with $h(-3)$ neither equal nor opposite to $h(3)$. A conclusion with no work earns nothing.

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5. The point $(-2, 1)$ is on the graph of $xy^2 - x^3 = 6$. (3 points)

(a) Test the graph for symmetry about the x-axis, the y-axis and the origin. For each symmetry it lacks, use a partner of $(-2, 1)$ to prove it.

(b) Name one more point on the graph, and explain why the graph is not the graph of a function.

ANSWER

(a) Symmetric about the x-axis only. (b) $(-2, -1)$; it lies on the same vertical line as $(-2, 1)$.

SOLUTION

Check the point: $(-2)(1)^2 - (-2)^3 = -2 + 8 = 6$.

(a) Replacing y by $-y$: $x(-y)^2 - x^3 = xy^2 - x^3$, the same equation. Symmetric about the x-axis.

Replacing x by $-x$ gives $-xy^2 + x^3 = 6$, a different equation. The partner $(2, 1)$ gives $2 - 8 = -6 \neq 6$. Not symmetric about the y-axis.

Replacing both gives $-xy^2 + x^3 = 6$ again. The partner $(2, -1)$ gives $2 \cdot 1 - 8 = -6 \neq 6$. Not symmetric about the origin.

(b) By the x-axis symmetry, $(-2, -1)$ is on the graph. The points $(-2, 1)$ and $(-2, -1)$ lie on the vertical line $x = -2$, so the graph fails the vertical line test and is not the graph of a function.

SCORING

1 point for the x-axis test done by substituting $-y$. 1 point for ruling out the y-axis and the origin, each with a partner point (or a correct argument that the new equation has different solutions). 1 point for $(-2, -1)$ with the vertical-line reason.

6. Isaac copied this table for a function whose graph is symmetric about the origin. Exactly one entry was copied wrong. (3 points)

x	−3	−2	−1	0	1	2	3
$f(x)$	8	−1	4	1	−4	1	−8

(a) Which entry is wrong, and what should it be?

(b) Could the table instead belong to a function symmetric about the vertical axis, with only one entry copied wrong? Explain.

ANSWER

(a) The entry at $x = 0$: it should be $f(0) = 0$.

(b) No: three pairs of opposite inputs disagree, and one corrected entry can fix only one pair.

SOLUTION

(a) Each point (a, b) needs the partner $(-a, -b)$. The pairs $f(-3) = 8$ and $f(3) = -8$, $f(-2) = -1$ and $f(2) = 1$, $f(-1) = 4$ and $f(1) = -4$ are all opposites, so they fit. But a function's graph symmetric about the origin passes through the origin: its point above $x = 0$ is its own partner, so $f(0) = -f(0)$ and $f(0) = 0$. The wrong entry is $f(0) = 1$, and it should be $f(0) = 0$.

(b) Symmetry about the vertical axis needs equal outputs at a and $-a$. But $8 \neq -8$, $-1 \neq 1$ and $4 \neq -4$: three pairs disagree. Changing one entry changes only one pair, so at least three entries would have to be wrong.

SCORING

1 point for naming the entry at $x = 0$; 1 point for the correct value 0 with a reason (the graph must pass through the origin, or the other pairs all fit); 1 point for "no" with the reason that three pairs disagree.

7. Here is a table for a function g with a point above every x. (3 points)

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x	-1	0	1	2	3	4	5
$g(x)$	6	1	-2	-3	-2	1	6

(a) Use the table to show that the graph of g is not symmetric about the vertical axis.

(b) Find the number h for which the graph of $y = g(x + h)$ is symmetric about the vertical axis, as far as the table shows, and give the table of $g(x + h)$ for $x = -3$ to 3.

(c) Can a vertical shift, $y = g(x) + c$, make the graph symmetric about the vertical axis? Explain.

ANSWER

(a) The partner of $(1, -2)$ would be $(-1, -2)$, but $g(-1) = 6$.

(b) $h = 2$:

x	-3	-2	-1	0	1	2	3
$g(x + 2)$	6	1	-2	-3	-2	1	6

(c) No: $g(1) + c$ and $g(-1) + c$ still differ by 8 for every c.

SOLUTION

(a) For symmetry about the vertical axis, $(1, -2)$ needs the partner $(-1, -2)$. The table gives $g(-1) = 6$, so the partner is missing.

(b) The table matches itself about $x = 2$: $g(1) = g(3) = -2$, $g(0) = g(4) = 1$, $g(-1) = g(5) = 6$. Moving the graph 2 units left brings that mirror line onto the y-axis, and $y = g(x + 2)$ is that move, so $h = 2$. Its values at $x = -3, \ldots, 3$ are $g(-1), \ldots, g(5)$, the row 6, 1, -2, -3, -2, 1, 6, which reads the same from both ends.

(c) No. A vertical shift adds the same c to every height. Then $g(1) + c = -2 + c$ and $g(-1) + c = 6 + c$ still differ by 8, so $(1, -2 + c)$ still has no partner. A vertical shift cannot move a mirror line sideways.

SCORING

1 point for a partner that is missing, from the table; 1 point for $h = 2$ with the correct table; 1 point for "no" with the reason that the heights at 1 and -1 still differ by 8 (or any pair that still disagrees).

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