Algebra 2 Unit 1 quiz 5: Linear functions in context, Domain and range from a graph (answer key)

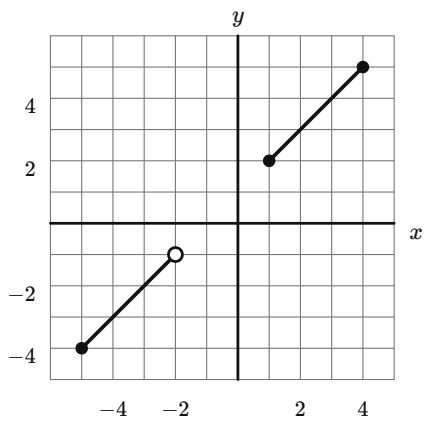
8 questions, 21 points

ANSWERS AT A GLANCE

1. Domain $[-5,-2)\cup[1,4]$; range $[-4,-1)\cup[2,5]$.
2. Domain $(-\infty,3)$; range $[-2,\infty)$. The open dot removes 3 from the domain but not 2 from the range.
3. Domain: all real numbers, $(-\infty,\infty)$. Range $-1<y<3$, that is $(-1,3)$.
4. (a) 145 cm is the depth on the day of the last snowfall; −2.5 cm per day: the snow loses 2.5 cm each day. (b) It asks after how many days the snow is 100 cm deep: 18 days. (c) After 58 days.
5. (a) $C(n)=21n+140$; the 140 dollars are the fixed charge every party pays, whatever the number of guests. (b) 56 guests
6. (a) A is ahead by 450 pages; B prints 15 pages per minute faster. (b) At 30 minutes after noon (12:30), with 1800 pages each. (c) B, by 225 pages.
7. (a) $A(k)=0.15k+18$ and $B(k)=0.18k+12$. (b) Plan A adds 6 dollars to every bill and plan B adds $0.03k$ dollars, which equals 6 at $k=200$; plan A gives the higher bill for uses below 200 kWh a month.
8. (a) Domain $(0,10]$; range $\{6,9,15\}$. (b) A parcel can weigh anything more than 0 kg and up to 10 kg; the only charges are 6, 9 and 15 dollars. Each piece is flat, so the graph jumps between heights and never passes the heights in between.

WORKED SOLUTIONS

1. Write the domain and range of the graph in the figure in interval notation. One piece runs from the closed dot (−5, −4) to the open dot (−2, −1); the other runs from the closed dot (1, 2) to the closed dot (4, 5). (2 points)



ANSWER

Domain $[-5,-2)\cup[1,4]$; range $[-4,-1)\cup[2,5]$.

SOLUTION

Domain. The first piece covers $-5\le x<-2$ and the second $1\le x\le 4$. Nothing lies above the stretch from −2 to 1:

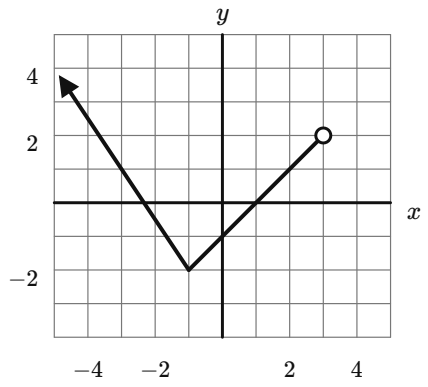
$$[-5,-2)\cup[1,4].$$

Range. The first piece rises from −4 toward −1, never reaching −1: $[-4,-1)$. The second rises from 2 to 5, both reached: $[2,5]$. No point of either piece has a height between −1 and 2, so the shadows do not overlap and the range has a gap too:

$$[-4,-1)\cup[2,5].$$

2. The graph in the figure is made of two straight pieces that meet at (−1, −2). The left piece falls from the upper left, 3 down for every 2 across, and its arrow says it carries on forever. The right piece rises, 1 up for every 1 across, to the open dot (3, 2). Find the domain and range in interval notation. Explain what the open dot does to each. (2 points)

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ANSWER

Domain $(-\infty, 3)$; range $[-2, \infty)$. The open dot removes 3 from the domain but not 2 from the range.

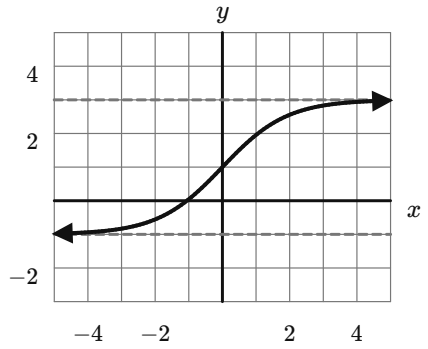
SOLUTION

Domain. The arrow carries the graph left forever, and on the right it stops at the open dot. Nothing else is above $x = 3$, so 3 is left out: $(-\infty, 3)$.

Range. The lowest point is the corner (−1, −2), which is on the graph. Going left, the left piece climbs without end, so every height from −2 up is reached: $[-2, \infty)$.

The open dot does not remove 2 from the range: the left piece passes height 2 at $x = -\dfrac{11}{3}$, about −3.7, since $-2 + \dfrac{3}{2} \cdot \dfrac{8}{3} = 2$.

3. The curve in the figure rises from left to right, with arrows at both ends. Going left it flattens out toward the dashed line $y = -1$, and going right toward the dashed line $y = 3$, never reaching either. Find the domain and range, as inequalities and in interval notation. (2 points)



ANSWER

Domain: all real numbers, $(-\infty, \infty)$. Range $-1 < y < 3$, that is (−1, 3).

SOLUTION

Domain. Both arrows carry the curve sideways forever, so every x is used: $(-\infty, \infty)$.

Range. The curve never touches either dashed line, so no point has height −1 or 3. It comes as close to each as you like, and since it is unbroken it passes every height between. Both ends of the range are open, though there is no open dot anywhere on the picture:

$$-1 < y < 3, \quad \text{that is} \quad (-1, 3).$$

4. After the last snowfall of the winter, the snow at a ski resort is $S(d) = 145 - 2.5d$ centimeters deep, d days later. (3 points)

(a) Say what 145 and −2.5 mean, with their units.

(b) What does the equation $S(d) = 100$ ask? Solve it.

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(c) When will the snow be gone, if the model holds?

ANSWER
(a) 145 cm is the depth on the day of the last snowfall; −2.5 cm per day: the snow loses 2.5 cm each day. (b) It asks after how many days the snow is 100 cm deep: 18 days. (c) After 58 days.

SOLUTION
(a) $S(0) = 145$, so the snow was 145 cm deep on the day of the last snowfall. The rate is −2.5 cm per day: the depth goes down by 2.5 cm every day.

(b) $S(d) = 100$ asks which day the depth is 100 cm. $145 - 2.5d = 100$, so $2.5d = 45$ and $d = 18$ days.

(c) The snow is gone when $S(d) = 0$: $2.5d = 145$, so $d = 58$ days.

SCORING
1 point for both meanings with units (the depth at the last snowfall, and 2.5 cm lost per day); 1 point for saying what $S(d) = 100$ asks and $d = 18$; 1 point for 58 days.

5. A hall prices a party by the number of guests. A party for 17 guests costs 497 dollars, and each guest more or fewer adds or takes off 21 dollars. (3 points)

(a) Write the cost $C(n)$, in dollars, of a party for n guests, and say what its starting value means.

(b) How many guests can come to a party that costs 1316 dollars?

ANSWER
(a) $C(n) = 21n + 140$; the 140 dollars are the fixed charge every party pays, whatever the number of guests. (b) 56 guests

SOLUTION
(a) The rate is 21 dollars per guest. Walking back from 17 guests to 0 takes off 17 × 21 = 357 dollars, so the starting value is 497 − 357 = 140 dollars and

$$C(n) = 21n + 140$$

No party has 0 guests, so $C(0) = 140$ is not the price of a real party; it is the fixed part of every price, such as the charge for the room.

(b) Solve $21n + 140 = 1316$: $21n = 1176$, so $n = 56$ guests.

SCORING
1 point for $C(n) = 21n + 140$; 1 point for saying 140 dollars is a fixed charge paid whatever the number of guests; 1 point for 56 guests.

6. Two printers in a print shop run at steady speeds. By t minutes after noon, printer A has printed $A(t) = 600 + 40t$ pages and printer B has printed $B(t) = 150 + 55t$ pages. (3 points)

(a) Which printer is ahead at noon, and by how many pages? Which prints faster, and by how much?

(b) When have they printed the same number of pages, and how many is that?

(c) Without finding $A(45)$ or $B(45)$, say which printer has printed more at 45 minutes after noon, and by how many pages.

ANSWER
(a) A is ahead by 450 pages; B prints 15 pages per minute faster. (b) At 30 minutes after noon (12:30), with 1800 pages each. (c) B, by 225 pages.

SOLUTION
(a) The starting values are 600 and 150 pages, so A is 450 pages ahead at noon. The rates are 40 and 55 pages per minute, so B prints 15 pages per minute faster.

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(b) B closes the 450 page gap at 15 pages a minute, so the gap is gone after $\frac{450}{15} = 30$ minutes. Check by solving $600 + 40t = 150 + 55t$: $450 = 15t$, $t = 30$. Then $A(30) = 600 + 1200 = 1800$ and $B(30) = 150 + 1650 = 1800$ pages.

(c) After they meet at 30 minutes, B keeps gaining 15 pages a minute. In the 15 minutes from 30 to 45 it gains 15 × 15 = 225 pages, so B is 225 pages ahead.

SCORING

1 point for A ahead by 450 pages and B faster by 15 pages per minute; 1 point for 30 minutes after noon and 1800 pages; 1 point for B ahead by 225 pages, reasoned from the rates.

7. A power company bills a home $E(k) = 0.15k + 12$ dollars for a month in which it uses k kilowatt-hours (kWh) of electricity. Next year the company will make one of two changes.

Plan A: the fixed monthly charge rises to 18 dollars, and the price per kWh stays the same.

Plan B: the price per kWh rises to 0.18 dollars, and the fixed charge stays the same. (3 points)

(a) Write the monthly bill under each plan, $A(k)$ and $B(k)$.

(b) For which monthly uses does plan A give the higher bill? Explain using how much each change adds to the bill.

ANSWER

(a) $A(k) = 0.15k + 18$ and $B(k) = 0.18k + 12$. (b) Plan A adds 6 dollars to every bill and plan B adds $0.03k$ dollars, which equals 6 at $k = 200$; plan A gives the higher bill for uses below 200 kWh a month.

SOLUTION

(a) Plan A changes only the starting value: $A(k) = 0.15k + 18$. Plan B changes only the rate: $B(k) = 0.18k + 12$.

(b) Compared with $E(k)$, plan A adds 18 − 12 = 6 dollars to every bill, whatever k is. Plan B adds 0.03 dollars for each kWh, $0.03k$ dollars in all, which grows with the use. The two add the same when $0.03k = 6$, at $k = 200$ kWh. For a home using less than 200 kWh, $0.03k < 6$, so plan A gives the higher bill. (Check at $k = 100$: $A(100) = 33$ and $B(100) = 30$.)

SCORING

1 point for both rules; 1 point for comparing the added amounts (6 dollars against $0.03k$ dollars) or solving $A(k) = B(k)$ to get $k = 200$; 1 point for plan A higher below 200 kWh.

8. A courier charges 6 dollars to send a parcel weighing up to 2 kilograms, 9 dollars for more than 2 kg and up to 5 kg, and 15 dollars for more than 5 kg and up to 10 kg. It does not take parcels over 10 kg. Samir graphs the charge C in dollars for a parcel of w kilograms as three horizontal segments: from the open dot (0, 6) to the closed dot (2, 6), from the open dot (2, 9) to the closed dot (5, 9), and from the open dot (5, 15) to the closed dot (10, 15). (3 points)

(a) Find the domain and range.

(b) Say what each means for the courier, and explain why the range is not an interval.

ANSWER

(a) Domain $(0, 10]$; range $\{6, 9, 15\}$. (b) A parcel can weigh anything more than 0 kg and up to 10 kg; the only charges are 6, 9 and 15 dollars. Each piece is flat, so the graph jumps between heights and never passes the heights in between.

SOLUTION

(a) *Domain.* The first piece starts just after $w = 0$ (open dot) and the pieces join with no gap: at $w = 2$ the closed dot (2, 6) supplies the input that the open dot (2, 9) leaves out, and likewise at $w = 5$. The last piece

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ends at the closed dot $w = 10$:

$$0 < w \le 10, \quad \text{that is} \quad (0, 10].$$

Range. Every point of the first piece has height 6, of the second 9, of the third 15. The range is the set of three numbers $\{6, 9, 15\}$.

(b) The domain says the courier takes a parcel of any weight above 0 kg and up to 10 kg. The range says every charge is 6, 9 or 15 dollars. The graph is not one unbroken piece: it jumps from height 6 to height 9 at $w = 2$ and from 9 to 15 at $w = 5$, with no point at any height in between, so no charge such as 12 dollars is ever made.

SCORING

1 point: domain $(0, 10]$, with 0 left out and 10 included. 1 point: range $\{6, 9, 15\}$ (an interval such as $[6, 15]$ earns 0). 1 point: both meanings stated in the courier's terms and the gaps in the range explained by the jumps.

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