Algebra 2 Unit 1 test: Parent functions and transformations (answer key)

12 questions, 46 points

ANSWERS AT A GLANCE

1. (a) The closed dot is $(-1, 5)$ and the open dot is $(5, 5)$. (b) Domain $[-1, 5)$; range $[-4, 5]$.
2. (a) $g(x) = \dfrac{3}{x}$: $y = \dfrac{1}{x}$ stretched vertically by a factor of 3. (b) $\left(\sqrt{3}, \sqrt{3}\right)$ and $\left(-\sqrt{3}, -\sqrt{3}\right)$
3. (a) $m(x) = -2\,f(x+3) - 4$: $a = -2$, $b = 1$, $h = -3$, $k = -4$. (b) $(-1, 6)$ both ways.
4. (a) Domain all real numbers; range $y \geq 0$. (b) Falls for $x \leq 0$, rises for $x \geq 0$; lowest point $(0, 0)$. (c) $x = -8$ and $x = 4$. (d) No: the points $(-2, 4)$ and $(2, 8)$ are not at the same height.
5. (a) $x = 1$ only. (b) $x < 0$ or $x > 1$
6. (a) $x = 4$ and $y = 5$ (b) Domain: all real numbers except 4; range: all real numbers except 5 (c) x-intercept $\left(\dfrac{19}{5}, 0\right)$, y-intercept $\left(0, \dfrac{19}{4}\right)$
7. (a) $g(x) = -(x-2)^3 - 1$; $(2, 1) \mapsto (2, -1)$; x-intercept $(1, 0)$, y-intercept $(0, 7)$.
   (b) $h(x) = (-x-2)^3 + 1 = -(x+2)^3 + 1$; $(2, 1) \mapsto (-2, 1)$; x-intercept $(-1, 0)$, y-intercept $(0, -7)$.
8. (a) $(1, -4)$, $(3, 0)$, $(1, 2)$ and $(-1, 0)$ (b) 12 square units
9. (a) 1.80 dollars per km on all three intervals; the rate is the same everywhere, so the table fits a linear function. (b) $F(d) = 1.8d + 5.7$: each kilometer adds 1.80 dollars, and every ride pays a fixed 5.70 dollars. (c) 23 km
10. (a) $y = -2(x-2)^2 + 5$ (b) Her rule gives -11 at $x = 4$, not -3; $b = \sqrt{2}$ or $b = -\sqrt{2}$ (c) $\big(b(x-2)\big)^2 = b^2(x-2)^2$, so b and $-b$ give the same rule and the same graph; with $b = \sqrt{2}$: $-2(-2)^2 + 5 = -3$
11. (a) $w(0) = 1$, not 0.
    (b) The center is $(1, -2)$.
    (c) $h = 1$ and $c = 2$:

x	−3	−2	−1	0	1	2	3
$m(x)$	−15	0	3	0	−3	0	15

12. (a) $(-1, 1) \mapsto \left(-\dfrac{1}{2}, 2\right)$, $(0, 0) \mapsto (0, 3)$, $(2, 4) \mapsto (1, -1)$; the graph is a downward arch from the closed dot $\left(-\dfrac{1}{2}, 2\right)$ up to $(0, 3)$ and down to the closed dot $(1, -1)$. (b) Domain $\left[-\dfrac{1}{2}, 1\right]$, range $[-1, 3]$. (c) The highest point of g is $(0, 3)$, which is not an endpoint, so the range reaches 3; the endpoint heights alone would give only $[-1, 2]$.

WORKED SOLUTIONS

1. The graph of $g(x) = x^2 - 4x$ is drawn only from a closed dot at $x = -1$ to an open dot at $x = 5$. Its lowest point is $(2, -4)$. (3 points)

(a) Use the rule to find the coordinates of both end dots.

(b) Find the domain and range of the graph in interval notation.

ANSWER

(a) The closed dot is $(-1, 5)$ and the open dot is $(5, 5)$. (b) Domain $[-1, 5)$; range $[-4, 5]$.

SOLUTION

(a) $g(-1) = (-1)^2 - 4(-1) = 1 + 4 = 5$ and $g(5) = 25 - 20 = 5$. The ends are the closed dot $(-1, 5)$ and the open dot $(5, 5)$.

(b) *Domain.* From $x = -1$, included, to $x = 5$, left out: $[-1, 5)$.

Range. The lowest point is the turning point, height -4, on the graph. Both ends are at height 5, the highest the graph reaches. The open dot misses height 5, but the closed dot $(-1, 5)$ supplies it. The left arm falls unbroken from 5 to -4, so every height between is reached:

$$-4 \leq y \leq 5, \quad \text{that is} \quad [-4, 5].$$

SCORING

1 point: both end points correct from the rule, with the open one named. 1 point: domain $[-1, 5)$. 1 point: range $[-4, 5]$, with 5 included because the closed dot reaches it.

2. The graph of g is a vertical stretch or compression of $y = \dfrac{1}{x}$ and passes through $\left(4, \dfrac{3}{4}\right)$. (3 points)

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(a) Write the rule for g and name the move.

(b) Find every point where the graph of g meets the line $y = x$. Give exact coordinates.

ANSWER

(a) $g(x) = \dfrac{3}{x}$: $y = \dfrac{1}{x}$ stretched vertically by a factor of 3. (b) $\left(\sqrt{3}, \sqrt{3}\right)$ and $\left(-\sqrt{3}, -\sqrt{3}\right)$

SOLUTION

(a) A vertical move multiplies the output: $g(x) = a \cdot \dfrac{1}{x} = \dfrac{a}{x}$. The point gives $\dfrac{a}{4} = \dfrac{3}{4}$, so $a = 4 \cdot \dfrac{3}{4} = 3$. Since 3 > 1, $g(x) = \dfrac{3}{x}$ is $y = \dfrac{1}{x}$ stretched vertically by a factor of 3.

(b) On the line $y = x$, so solve $\dfrac{3}{x} = x$ with $x \neq 0$: $x^2 = 3$, so $x = \sqrt{3}$ or $x = -\sqrt{3}$. Each point has $y = x$: $\left(\sqrt{3}, \sqrt{3}\right)$ and $\left(-\sqrt{3}, -\sqrt{3}\right)$. The parent meets $y = x$ at (1, 1) and (−1, −1); the stretch moves those points straight up and down, off the line, so the crossings move.

SCORING

1 point: $g(x) = \dfrac{3}{x}$ named as a vertical stretch by 3. 1 point: the equation $\dfrac{3}{x} = x$, giving $x^2 = 3$. 1 point: both points with exact coordinates.

3. Let $g(x) = 2\,f(x - 1) + 3$ and $m(x) = -g(x + 4) - 1$. (3 points)

(a) Write $m(x)$ in the form $a\,f\big(b(x - h)\big) + k$ and give a, b, h and k.

(b) The point (2, −5) is on the graph of f. Find the point of m that comes from it twice: once by going through g, and once by using your rule from (a).

ANSWER

(a) $m(x) = -2\,f(x + 3) - 4$: $a = -2$, $b = 1$, $h = -3$, $k = -4$. (b) (−1, 6) both ways.

SOLUTION

(a) Replace x by $x + 4$ in g: $g(x + 4) = 2\,f(x + 4 - 1) + 3 = 2\,f(x + 3) + 3$. Then

$$m(x) = -\big(2\,f(x + 3) + 3\big) - 1 = -2\,f(x + 3) - 4$$

So $a = -2$, $b = 1$, $h = -3$, $k = -4$.

(b) Through g: g sends (p, q) to $(p + 1,\ 2q + 3)$, so $(2, -5) \mapsto (3, -7)$. Then m moves a point of g by $(P, Q) \mapsto (P - 4,\ -Q - 1)$, so $(3, -7) \mapsto (-1, 6)$.

With the rule from (a): $(p, q) \mapsto (p - 3,\ -2q - 4)$, so $(2, -5) \mapsto (-1,\ 10 - 4) = (-1, 6)$. Check: $m(-1) = -2\,f(2) - 4 = 10 - 4 = 6$.

SCORING

1 point for $m(x) = -2\,f(x + 3) - 4$ with a, b, h, k; 1 point for (−1, 6) through g, with (3, −7) shown; 1 point for (−1, 6) from the rule in (a).

4. Diego draws one curve from two parent graphs: for $x \leq 0$ it follows $y = x^2$, and for $x > 0$ it follows $y = x^3$. (4 points)

(a) Give the domain and range of his curve.

(b) Where does the curve fall and where does it rise? What is its lowest point?

(c) Find every input whose output is 64.

(d) Is the curve its own mirror image in the y-axis? Use the inputs −2 and 2 to explain.

ANSWER

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(a) Domain all real numbers; range $y \geq 0$. (b) Falls for $x \leq 0$, rises for $x \geq 0$; lowest point (0, 0). (c) $x = -8$ and $x = 4$. (d) No: the points (−2, 4) and (2, 8) are not at the same height.

SOLUTION

(a) Every input uses one of the two rules, so the domain is all real numbers. The left piece gives the squares of the numbers $x \leq 0$, which are all the heights $y \geq 0$. The right piece gives the cubes of positive numbers, all positive heights. No output is negative, since the cubing rule is used only for positive inputs. Together the range is $y \geq 0$.

(b) The left half of the U falls to (0, 0), and the right half of the cubic rises from there. So the curve falls for $x \leq 0$, rises for $x \geq 0$, and its lowest point is (0, 0).

(c) Left piece: $x^2 = 64$ with $x \leq 0$ gives $x = -8$; the input 8 belongs to the other piece. Right piece: $x^3 = 64$ gives $x = 4$, which is positive. So the inputs are $x = -8$ and $x = 4$.

(d) At $x = -2$ the curve is at $(-2)^2 = 4$, and at $x = 2$ it is at $2^3 = 8$. A curve that is its own mirror image in the y-axis has each point (a, b) matched by $(-a, b)$ at the same height, so this curve is not.

SCORING

1 point for each part: (a) both the domain and the range $y \geq 0$; (b) where it falls and rises and the lowest point (0, 0); (c) both inputs −8 and 4, no others; (d) "no" with the two heights 4 and 8.

5. Compare the parent graphs $y = |x|$ and $y = \dfrac{1}{x}$. (4 points)

(a) Find every x with $|x| = \dfrac{1}{x}$.

(b) Find every $x \neq 0$ with $|x| > \dfrac{1}{x}$.

Treat $x < 0$ and $x > 0$ separately.

ANSWER

(a) $x = 1$ only. (b) $x < 0$ or $x > 1$

SOLUTION

For $x < 0$: $|x|$ is positive and $\dfrac{1}{x}$ is negative, so $|x| > \dfrac{1}{x}$ for every such x, and they are never equal. The V is in the second quadrant there, the left piece of the reciprocal graph in the third.

For $x > 0$: $|x| = x$. The equation $x = \dfrac{1}{x}$ gives $x^2 = 1$, so $x = 1$ (the root −1 is not positive). The inequality $x > \dfrac{1}{x}$, multiplied by the positive number x, gives $x^2 > 1$, so $x > 1$. For $0 < x < 1$, $x < 1 < \dfrac{1}{x}$, so the reciprocal is higher.

(a) $x = 1$ only, at the point (1, 1). (b) $x < 0$ or $x > 1$.

SCORING

1 point for showing every $x < 0$ works in (b) because $\dfrac{1}{x}$ is negative there; 1 point for reducing the case $x > 0$ to $x^2 = 1$ or $x^2 > 1$; 1 point for (a) $x = 1$ only; 1 point for (b) $x < 0$ or $x > 1$.

6. Let $g(x) = \dfrac{1}{x - 4} + 5$. (4 points)

(a) Give the asymptotes of the graph of g.

(b) Give the domain and the range of g.

(c) Find the x-intercept and the y-intercept of the graph.

ANSWER

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(a) $x = 4$ and $y = 5$ (b) Domain: all real numbers except 4; range: all real numbers except 5 (c) x-intercept $\left(\frac{19}{5}, 0\right)$, y-intercept $\left(0, \frac{19}{4}\right)$

SOLUTION

$g(x) = f(x-4) + 5$ with $f(x) = \frac{1}{x}$, so the graph is $y = \frac{1}{x}$ moved by $h = 4$ across and $k = 5$ up.

(a) The parent's asymptotes $x = 0$ and $y = 0$ move with the graph: $x = 4$, where $x - 4 = 0$, and $y = 5$, because $\frac{1}{x-4}$ is never 0.

(b) The only forbidden input is 4, and the only output never reached is 5: domain all real numbers except 4, range all real numbers except 5.

(c) Set $g(x) = 0$: $\frac{1}{x-4} = -5$, so $x - 4 = -\frac{1}{5}$ and $x = \frac{19}{5}$. At $x = 0$: $g(0) = \frac{1}{-4} + 5 = \frac{19}{4}$.

SCORING

1 point: both asymptotes. 1 point: domain and range, each excluding the right number. 1 point: the x-intercept $\left(\frac{19}{5}, 0\right)$ with work. 1 point: the y-intercept $\left(0, \frac{19}{4}\right)$.

7. Let $f(x) = (x-2)^3 + 1$. Its graph is $y = x^3$ shifted so that the point (0, 0) of the parent lands at (2, 1). (4 points)

(a) For $g(x) = -f(x)$: write the rule, say where the point (2, 1) goes, and find the x-intercept and the y-intercept.

(b) For $h(x) = f(-x)$: write the rule, say where the point (2, 1) goes, and find the x-intercept and the y-intercept.

ANSWER

(a) $g(x) = -(x-2)^3 - 1$; $(2, 1) \mapsto (2, -1)$; x-intercept (1, 0), y-intercept (0, 7).

(b) $h(x) = (-x-2)^3 + 1 = -(x+2)^3 + 1$; $(2, 1) \mapsto (-2, 1)$; x-intercept (−1, 0), y-intercept (0, −7).

SOLUTION

First the intercepts of f. $f(0) = (-2)^3 + 1 = -7$, so the y-intercept is (0, −7). $f(x) = 0$ when $(x-2)^3 = -1$, so $x - 2 = -1$ and $x = 1$: the x-intercept is (1, 0).

(a) The minus sign reaches the whole output: $g(x) = -\left[(x-2)^3 + 1\right] = -(x-2)^3 - 1$. Each point (a, b) goes to $(a, -b)$, so (2, 1) goes to (2, −1). The zero of f stays: x-intercept (1, 0), since $g(1) = -0 = 0$. The y-intercept changes sign: $g(0) = -f(0) = 7$, so (0, 7).

(b) Every x is replaced by $-x$: $h(x) = (-x-2)^3 + 1$. Since $-x - 2 = -(x+2)$ and a cube keeps the sign, $h(x) = -(x+2)^3 + 1$. Each point (a, b) goes to $(-a, b)$, so (2, 1) goes to (−2, 1). The x-intercept moves across: (−1, 0); check $h(-1) = (1-2)^3 + 1 = 0$. The y-intercept is on the mirror and stays: $h(0) = f(0) = -7$, so (0, −7).

SCORING

1 point for $g(x) = -(x-2)^3 - 1$ with (2, −1); 1 point for both intercepts of g, (1, 0) and (0, 7); 1 point for $h(x) = (-x-2)^3 + 1$ or an equal form with (−2, 1); 1 point for both intercepts of h, (−1, 0) and (0, −7).

8. The graphs of $y = 2|x-1| - 4$ and $y = -|x-1| + 2$ enclose a region with four corners. (4 points)

(a) Find the four corners.

(b) Find the area of the region.

ANSWER

(a) (1, −4), (3, 0), (1, 2) and (−1, 0) (b) 12 square units

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SOLUTION

(a) Both V's have axis $x = 1$. The first opens upward with vertex (1, −4); the second opens downward with vertex (1, 2). These two vertices are two corners.

The other corners are where the graphs meet: $2|x - 1| - 4 = -|x - 1| + 2$ gives $3|x - 1| = 6$, so $|x - 1| = 2$ and $x = -1$ or $x = 3$. The height there is −2 + 2 = 0, so the corners are (−1, 0) and (3, 0).

(b) The region is a kite. Its diagonals are the vertical segment from (1, −4) to (1, 2), length 6, and the horizontal segment from (−1, 0) to (3, 0), length 4, and they are perpendicular. The area is $\frac{1}{2} \cdot 6 \cdot 4 = 12$ square units. (Or: two triangles on the base from (−1, 0) to (3, 0), with heights 2 and 4: $\frac{1}{2} \cdot 4 \cdot 2 + \frac{1}{2} \cdot 4 \cdot 4 = 4 + 8 = 12$.)

SCORING

1 point: both vertices (1, −4) and (1, 2). 1 point: sets the rules equal and gets $|x - 1| = 2$. 1 point: the corners (−1, 0) and (3, 0). 1 point: area 12 (follow through from the corners found).

9. A ride-share app shows the fare $F(d)$, in dollars, for a ride of d kilometers. (4 points)

d (km)	3	7	12	20
$F(d)$ (dollars)	11.10	18.30	27.30	41.70

(a) Find the average rate of change of F over each interval between neighboring columns, and explain why the table fits a linear function.

(b) Write $F(d)$ and say what both of its numbers mean, with their units.

(c) How long a ride has a fare of 47.10 dollars?

ANSWER

(a) 1.80 dollars per km on all three intervals; the rate is the same everywhere, so the table fits a linear function. (b) $F(d) = 1.8d + 5.7$: each kilometer adds 1.80 dollars, and every ride pays a fixed 5.70 dollars. (c) 23 km

SOLUTION

(a) The steps in d are not equal, so divide each change in fare by its change in distance:

- 3 to 7: $\frac{18.30 - 11.10}{4} = \frac{7.20}{4} = 1.80$
- 7 to 12: $\frac{27.30 - 18.30}{5} = \frac{9.00}{5} = 1.80$
- 12 to 20: $\frac{41.70 - 27.30}{8} = \frac{14.40}{8} = 1.80$

The average rate of change is 1.80 dollars per km on every interval, so the fare changes by the same amount for each kilometer: linear.

(b) Walking back from 3 km to 0: $b = 11.10 - 3(1.80) = 11.10 - 5.40 = 5.70$. So $F(d) = 1.8d + 5.7$. Check: 1.8(20) + 5.7 = 36 + 5.7 = 41.7. Each kilometer adds 1.80 dollars, and $F(0) = 5.70$ dollars is the fixed part of every fare (a ride of 0 km is not a real ride).

(c) Solve $1.8d + 5.7 = 47.1$: $1.8d = 41.4$, so $d = 23$ km.

SCORING

1 point for the three rates, all 1.80, with the reason the table is linear; 1 point for $F(d) = 1.8d + 5.7$; 1 point for both meanings with units; 1 point for 23 km.

10. A parabola has its vertex at (2, 5) and passes through (4, −3). (4 points)

(a) Write its rule in the form $y = a(x - h)^2 + k$.

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(b) Sofia wants to write the rule with a horizontal compression instead, as $y = -\big(b(x-2)\big)^2 + 5$. She takes $b = 2$, "because a was 2 apart from its sign". Test her rule at (4, −3), and find every value of b that works.

(c) Explain why the graph cannot tell you the sign of b, and check one of your rules from (b) at the point (0, −3).

ANSWER

(a) $y = -2(x-2)^2 + 5$ (b) Her rule gives −11 at $x = 4$, not −3; $b = \sqrt{2}$ or $b = -\sqrt{2}$ (c) $\big(b(x-2)\big)^2 = b^2(x-2)^2$, so b and $-b$ give the same rule and the same graph; with $b = \sqrt{2}$: $-2(-2)^2 + 5 = -3$

SOLUTION

(a) The vertex gives $h = 2$, $k = 5$. With (4, −3): $a = \dfrac{-3-5}{(4-2)^2} = \dfrac{-8}{4} = -2$, so $y = -2(x-2)^2 + 5$.

(b) Sofia's rule at $x = 4$: $-\big(2 \cdot 2\big)^2 + 5 = -16 + 5 = -11$, not −3. A factor b inside the square comes out squared, $\big(b(x-2)\big)^2 = b^2(x-2)^2$, so the rule matches (a) when $b^2 = 2$. Directly from the point: $-(2b)^2 + 5 = -3$, so $4b^2 = 8$, $b^2 = 2$ and $b = \sqrt{2}$ or $b = -\sqrt{2}$.

(c) Both values give $b^2 = 2$, so both turn into the same rule $y = -2(x-2)^2 + 5$ and draw the same parabola; nothing in the picture can show which sign was used. Check $b = \sqrt{2}$ at (0, −3): $-\big(\sqrt{2}(-2)\big)^2 + 5 = -8 + 5 = -3$.

SCORING

1 point: $y = -2(x-2)^2 + 5$. 1 point: Sofia's rule tested, giving −11 instead of −3. 1 point: $b = \sqrt{2}$ and $b = -\sqrt{2}$, from $b^2 = 2$. 1 point: the reason (the square removes the sign of b) and the check at (0, −3).

11. Here is a table for a function w. Its graph turns onto itself under a half turn about some point, but that point is not the origin. (4 points)

x	−2	−1	0	1	2	3	4
$w(x)$	−17	−2	1	−2	−5	−2	13

(a) Use one entry to show that the graph of w is not symmetric about the origin.

(b) Pair up the entries to find the center of the half turn.

(c) Find numbers h and c so that the graph of $m(x) = w(x+h) + c$ is symmetric about the origin, and give the table of m for $x = -3$ to 3.

ANSWER

(a) $w(0) = 1$, not 0.

(b) The center is (1, −2).

(c) $h = 1$ and $c = 2$:

x	−3	−2	−1	0	1	2	3
$m(x)$	−15	0	3	0	−3	0	15

SOLUTION

(a) A function's graph symmetric about the origin passes through the origin, but $w(0) = 1$.

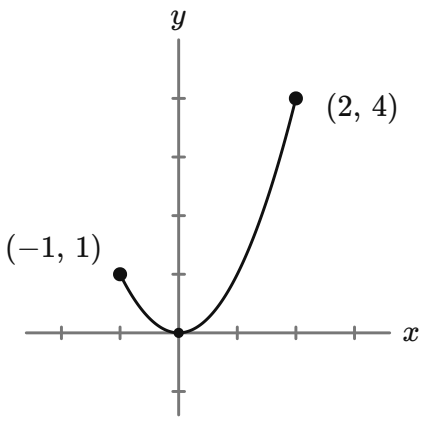
(b) Pair the ends inward: (−2, −17) with (4, 13), (−1, −2) with (3, −2), (0, 1) with (2, −5). Each pair has midpoint $\left(\dfrac{-2+4}{2}, \dfrac{-17+13}{2}\right) = (1, -2)$, and so on, and (1, −2) is on the graph, its own partner. The center is (1, −2).

(c) To make the center land on the origin, move the graph 1 left and 2 up: $m(x) = w(x+1) + 2$, so $h = 1$ and $c = 2$. Then $m(-3) = w(-2) + 2 = -15$, $m(-2) = w(-1) + 2 = 0$, and so on, giving the row −15, 0, 3, 0, −3, 0, 15. Check: each pair $m(-a)$, $m(a)$ is a pair of opposites, and $m(0) = 0$.

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SCORING
1 point for $w(0) = 1 \neq 0$ (or a pair of opposite inputs whose outputs are not opposites); 1 point for the center $(1, -2)$ from at least two pairs; 1 point for $h = 1$ and $c = 2$; 1 point for the correct table of m.

12. The figure shows the whole graph of a function f: the part of the parabola $y = x^2$ from the closed dot $(-1, 1)$ to the closed dot $(2, 4)$, passing through its vertex $(0, 0)$. Let $g(x) = 3 - f(2x)$. (5 points)



(a) Carry the two endpoints and the vertex to the graph of g, and sketch the graph of g.

(b) Give the domain and range of g in interval notation.

(c) Sofia says the range of g runs from the height of one endpoint of its graph to the height of the other. Explain why she is wrong.

ANSWER
(a) $(-1, 1) \mapsto \left(-\frac{1}{2}, 2\right)$, $(0, 0) \mapsto (0, 3)$, $(2, 4) \mapsto (1, -1)$; the graph is a downward arch from the closed dot $\left(-\frac{1}{2}, 2\right)$ up to $(0, 3)$ and down to the closed dot $(1, -1)$. (b) Domain $\left[-\frac{1}{2}, 1\right]$, range $[-1, 3]$. (c) The highest point of g is $(0, 3)$, which is not an endpoint, so the range reaches 3; the endpoint heights alone would give only $[-1, 2]$.

SOLUTION
(a) Write $g(x) = -f(2x) + 3$: $a = -1$, $b = 2$, $h = 0$, $k = 3$. Each point (p, q) goes to $\left(\frac{p}{2},\ 3 - q\right)$: $(-1, 1) \mapsto \left(-\frac{1}{2}, 2\right)$, $(0, 0) \mapsto (0, 3)$, $(2, 4) \mapsto (1, -1)$. The negative a turns the piece upside down, so the sketch is a downward arch: a closed dot at $\left(-\frac{1}{2}, 2\right)$, rising to its top at $(0, 3)$, then falling to a closed dot at $(1, -1)$, squeezed to half the parent's width by the compression. (In one formula, $g(x) = 3 - 4x^2$ for $-\frac{1}{2} \leq x \leq 1$; check: $3 - 4(1)^2 = -1$.)

(b) The domain of f is $[-1, 2]$; each input p goes to $\frac{p}{2}$, so the domain of g is $\left[-\frac{1}{2}, 1\right]$. The range of f is $[0, 4]$: its lowest point is the vertex, not an endpoint. Each height q goes to $3 - q$, which reverses order, so $[0, 4]$ becomes $[3 - 4, 3 - 0] = [-1, 3]$.

(c) The endpoint heights of g are 2 and -1, which would give $[-1, 2]$. But the vertex of f, inside its domain, carries to the highest point $(0, 3)$ of g, also inside. The range is carried from the whole range of f, $[0, 4]$, not from its endpoint heights 1 and 4, so it is $[-1, 3]$.

SCORING
1 point for the three image points; 1 point for a downward arch through them with closed dots at the ends; 1 point for the domain $\left[-\frac{1}{2}, 1\right]$; 1 point for the range $[-1, 3]$; 1 point for explaining that the top $(0, 3)$ comes from the vertex, not an endpoint.

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