Algebra 2 Unit 1 worksheet: Combining transformations and the general form (answer key)

ANSWERS AT A GLANCE

1. (a) $a=3, b=1, h=1, k=8$ (b) $a=1, b=2, h=-5, k=-4$ (c) $a=-1, b=-1, h=3, k=6$ (d) $a=-\frac{1}{2}, b=4, h=-3, k=0$
2. $x=-3$
3. (a) $y=(3x-2)^2=\left(3\left(x-\frac{2}{3}\right)\right)^2$, with vertex $\left(\frac{2}{3}, 0\right)$. (b) Right 6: $y=(3x-6)^2=\big(3(x-2)\big)^2$, with vertex $(2, 0)$.
4. (a) Her moves give (3, −7), then (3, −21). The true point is (3, 3). (b) Shift down 4: $3\big(f(x)-4\big)=3\,f(x)-12$.
5. (a) $a=2, b=-1, h=4, k=3$: reflect across the y-axis, then shift right 4; stretch vertically by 2, then shift up 3. (b) $(1,1)\mapsto(3,5)$ and $(-1,-1)\mapsto(5,1)$. (c) $y=\frac{-2}{x-4}+3$: $a=-2, h=4, k=3$; shift right 4; stretch vertically by 2 and reflect across the x-axis, then shift up 3. $(1,1)\mapsto(5,1)$, which is on the graph.
6. (a) Shift left 2; stretch vertically by a factor of 1.8, then shift up 32. (b) Low: $s=3$, 14 degrees Fahrenheit; high: $s=12$, 59 degrees Fahrenheit.

WORKED SOLUTIONS

1. Each rule is built from one of the parents $y=x^2$, $y=x^3$, $y=\sqrt{x}$ or $y=\frac{1}{x}$. Write it in the form $y=a\,f\big(b(x-h)\big)+k$ and give a, b, h and k.

(a) $y=\dfrac{3}{x-1}+8$ (b) $y=(2x+10)^3-4$ (c) $y=6-\sqrt{3-x}$ (d) $y=-\dfrac{1}{2}\sqrt{4x+12}$

ANSWER

(a) $a=3, b=1, h=1, k=8$ (b) $a=1, b=2, h=-5, k=-4$ (c) $a=-1, b=-1, h=3, k=6$ (d) $a=-\frac{1}{2}, b=4, h=-3, k=0$

SOLUTION

(a) Parent $\frac{1}{x}$: $\dfrac{3}{x-1}=3\cdot\dfrac{1}{x-1}$, so $a=3, b=1, h=1, k=8$.

(b) Parent x^3: factor the inside, $2x+10=2(x+5)$, so $y=\big(2(x+5)\big)^3-4$: $a=1, b=2, h=-5, k=-4$. The shift is 5 to the left, not 10.

(c) Parent $\sqrt{x}$: reorder as $y=-\sqrt{3-x}+6$ and factor −1 out of the inside, $3-x=-(x-3)$. So $a=-1$, $b=-1$, $h=3$, $k=6$.

(d) Parent $\sqrt{x}$: $4x+12=4(x+3)$, so $y=-\dfrac{1}{2}\sqrt{4(x+3)}+0$: $a=-\frac{1}{2}$, $b=4$, $h=-3$, $k=0$.

2. A function f takes the value −5 at exactly one input, $x=-19$. Let $g(x)=-2\,f(4x-7)+7$. Solve $g(x)=17$.

ANSWER

$x=-3$

SOLUTION

Undo the outside moves first:

$$-2\,f(4x-7)+7=17 \quad\Longrightarrow\quad -2\,f(4x-7)=10 \quad\Longrightarrow\quad f(4x-7)=-5$$

The only input at which f gives −5 is −19, so the inside must equal it: $4x-7=-19$, so $4x=-12$ and $x=-3$.

Check: $g(-3)=-2\,f(-19)+7=-2\cdot(-5)+7=17$. The solution is the point (−19, −5) of f carried to (−3, 17) on g.

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3. Ryan starts from $y = x^2$. He shifts the graph right 2, then compresses it horizontally by a factor of $\frac{1}{3}$ toward the y-axis.

(a) Write Ryan's rule in the form $y = \big(b(x - h)\big)^2$ and find where the vertex ends up.

(b) Ryan wanted the vertex at (2, 0). Keeping his order, a shift right first and then the same compression, how far right should he shift? Write that rule and check its vertex.

ANSWER

(a) $y = (3x - 2)^2 = \left(3\left(x - \frac{2}{3}\right)\right)^2$, with vertex $\left(\frac{2}{3}, 0\right)$. (b) Right 6: $y = (3x - 6)^2 = \big(3(x - 2)\big)^2$, with vertex (2, 0).

SOLUTION

(a) Shift right 2: $y = (x - 2)^2$. A horizontal compression by $\frac{1}{3}$ replaces x by $3x$: $y = (3x - 2)^2$. Factor the inside: $3x - 2 = 3\left(x - \frac{2}{3}\right)$, so $b = 3$ and $h = \frac{2}{3}$. The vertex (0, 0) goes right to (2, 0), and then the compression divides its x-coordinate by 3: $\left(\frac{2}{3}, 0\right)$. The compression came after the shift, so it pulled the shifted vertex toward the y-axis too.

(b) A shift right d followed by the compression gives $y = (3x - d)^2 = \left(3\left(x - \frac{d}{3}\right)\right)^2$, so $h = \frac{d}{3}$. For $h = 2$ he needs $d = 6$. The rule is $y = (3x - 6)^2 = \big(3(x - 2)\big)^2$. Check: at $x = 2$ the inside is 0, so the vertex is (2, 0).

4. Naomi describes $g(x) = 3\,f(x) - 12$ as "shift down 12, then stretch vertically by a factor of 3." The point (3, 5) is on the graph of f.

(a) Follow (3, 5) through Naomi's two moves in her order. Then find the point of g that really comes from (3, 5).

(b) Naomi wants to keep her order: a shift down first, then the stretch by 3. How far down must she shift so that the two moves give g? Show that your answer works.

ANSWER

(a) Her moves give (3, −7), then (3, −21). The true point is (3, 3). (b) Shift down 4: $3\big(f(x) - 4\big) = 3\,f(x) - 12$.

SOLUTION

(a) Naomi's way: down 12 takes (3, 5) to (3, −7); the stretch multiplies that height by 3, giving (3, −21). The rule itself says $g(3) = 3\,f(3) - 12 = 3 \cdot 5 - 12 = 3$, so the true point is (3, 3). Her order stretched the shift as well, making it a shift of 36 instead of 12.

(b) A shift down d followed by the stretch gives $3\big(f(x) - d\big) = 3\,f(x) - 3d$. For this to be $3\,f(x) - 12$ we need $3d = 12$, so $d = 4$. Check with the point: down 4 gives (3, 1), and the stretch gives (3, 3), the true point.

5. Let $y = \dfrac{2}{4 - x} + 3$, built from $y = \dfrac{1}{x}$.

(a) Write the rule as $a\,f\big(b(x - h)\big) + k$ with $b = -1$, and describe the moves in order.

(b) Carry the parent's points (1, 1) and (−1, −1) across with these moves.

(c) Show that the same rule can also be written with $b = 1$. Give its a, h and k, describe its moves, and carry (1, 1) across with them. Check that the point you get is on the graph.

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ANSWER

(a) $a = 2$, $b = -1$, $h = 4$, $k = 3$: reflect across the y-axis, then shift right 4; stretch vertically by 2, then shift up 3. (b) $(1, 1) \mapsto (3, 5)$ and $(-1, -1) \mapsto (5, 1)$. (c) $y = \dfrac{-2}{x - 4} + 3$: $a = -2$, $h = 4$, $k = 3$; shift right 4; stretch vertically by 2 and reflect across the x-axis, then shift up 3. $(1, 1) \mapsto (5, 1)$, which is on the graph.

SOLUTION

(a) Factor −1 out of the bottom: $4 - x = -(x - 4)$, so $y = 2 \cdot \dfrac{1}{-(x - 4)} + 3$. That gives $a = 2$, $b = -1$, $h = 4$, $k = 3$. Horizontally: reflect across the y-axis, then shift right 4. Vertically: stretch by a factor of 2, then shift up 3.

(b) A point goes to $(-p + 4,\ 2q + 3)$. So $(1, 1) \mapsto (3, 5)$ and $(-1, -1) \mapsto (5, 1)$. Check: $\dfrac{2}{4 - 3} + 3 = 5$ and $\dfrac{2}{4 - 5} + 3 = -2 + 3 = 1$.

(c) $\dfrac{2}{4 - x} = \dfrac{2}{-(x - 4)} = \dfrac{-2}{x - 4}$, so $y = -2 \cdot \dfrac{1}{x - 4} + 3$ with $a = -2$, $b = 1$, $h = 4$, $k = 3$. Moves: shift right 4; stretch vertically by 2 and reflect across the x-axis, then shift up 3. A point goes to $(p + 4,\ -2q + 3)$, so $(1, 1) \mapsto (5, 1)$. It is on the graph: it is the point (−1, −1) went to in (b). The two descriptions move the parent's points to different places, yet they make the same graph, because the reflection of $y = \dfrac{1}{x}$ across the y-axis is the same curve as its reflection across the x-axis.

6. Elena's weather station logs the air temperature as $C(t)$ degrees Celsius, t hours after midnight. On Monday the lowest reading was −10 degrees at $t = 5$ and the highest was 15 degrees at $t = 14$. Her cousin Arjun lives in a time zone 2 hours behind hers, reads temperatures in degrees Fahrenheit, and posts the same day's readings as

$$F(s) = 1.8\,C(s + 2) + 32,$$

where s is hours after midnight on his clock.

(a) Describe, in order, the moves that take the graph of C to the graph of F.

(b) At what time on Arjun's clock does the day's low appear, and what is its Fahrenheit reading? Do the same for the high.

ANSWER

(a) Shift left 2; stretch vertically by a factor of 1.8, then shift up 32. (b) Low: $s = 3$, 14 degrees Fahrenheit; high: $s = 12$, 59 degrees Fahrenheit.

SOLUTION

(a) Matching $a\,C\big(b(s - h)\big) + k$: $a = 1.8$, $b = 1$, $h = -2$, $k = 32$. Horizontally: shift left 2. Vertically: stretch by a factor of 1.8, then shift up 32. The horizontal shift can come at any point; the stretch must come before the shift up.

(b) Each point goes to $(p - 2,\ 1.8q + 32)$. The low (5, −10) goes to (3, −18 + 32) = (3, 14): at 3 a.m. on Arjun's clock, 14 degrees Fahrenheit. The high (14, 15) goes to (12, 27 + 32) = (12, 59): at noon, 59 degrees Fahrenheit. A clock 2 hours behind shows every reading 2 hours earlier, which is the shift left.

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