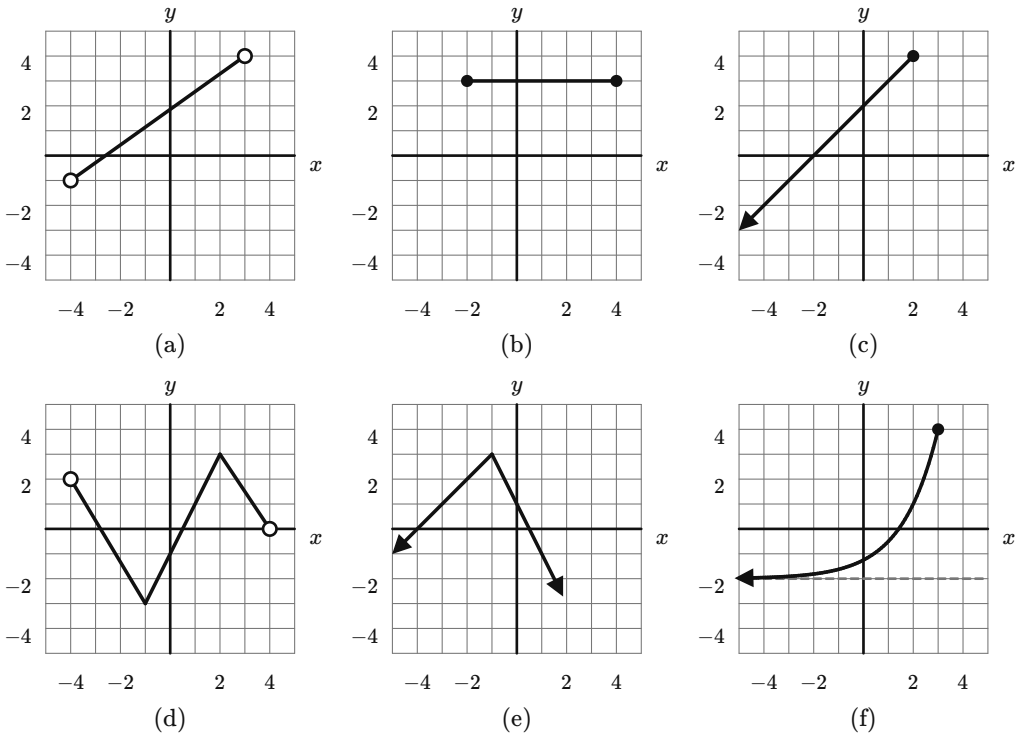
Algebra 2 Unit 1 worksheet: Domain and range from a graph (answer key)

ANSWERS AT A GLANCE

1. (a) domain $(-4, 3)$, range $(-1, 4)$ (b) domain $[-2, 4]$, range $\{3\}$ (c) domain $(-\infty, 2]$, range $(-\infty, 4]$ (d) domain $(-4, 4)$, range $[-3, 3]$ (e) domain $(-\infty, \infty)$, range $(-\infty, 3]$ (f) domain $(-\infty, 3]$, range $(-2, 4]$
2. (a) Yes: the closed dot $(1, 3)$ is above $x = 1$, so $f(1) = 3$. (b) No. (c) No: the segment already reaches height 3, at $(-2, 3)$. (d) Domain $[-3, 3]$; range $[-2, 0) \cup (0, 4]$.
3. The closed end is $(-3, 7)$ and the open end is $(5, -1)$; the segment falls.
4. (a) Domain $[-2, 4]$, range $[-5, 1]$. (b) Domain $[-2, 4]$, range $[-2, 1]$.
5. (a) Domain $0 \le t < 6$ or $9 \le t \le 18$, that is $[0, 6) \cup [9, 18]$. Range $-6 \le T < -2$ or $4 \le T \le 11$, that is $[-6, -2) \cup [4, 11]$. (b) No temperatures were logged from hour 6 until hour 9. (c) 0 is not in the range. Elena is right about the air, but the range holds only the logged temperatures, so it does not change.

WORKED SOLUTIONS

1. Write the domain and the range of each graph in the figure in interval notation. Every dot and corner sits on a grid point. In (f) the curve flattens out toward the dashed line $y = -2$ as it goes left and never reaches it.



ANSWER

(a) domain $(-4, 3)$, range $(-1, 4)$ (b) domain $[-2, 4]$, range $\{3\}$ (c) domain $(-\infty, 2]$, range $(-\infty, 4]$ (d) domain $(-4, 4)$, range $[-3, 3]$ (e) domain $(-\infty, \infty)$, range $(-\infty, 3]$ (f) domain $(-\infty, 3]$, range $(-2, 4]$

SOLUTION

(a) Both ends are open dots, at $x = -4$ and $x = 3$, so neither end is in the domain: $(-4, 3)$. The segment rises, so its lowest and highest heights are at those same open ends, -1 and 4, and no other point reaches them: $(-1, 4)$.

(b) The segment runs from $x = -2$ to $x = 4$, both closed: $[-2, 4]$. Every point has height 3, so the range is the single number $\{3\}$.

(c) The right end is the closed dot at $x = 2$ and the arrow carries the ray left forever: $(-\infty, 2]$. The highest point is the dot, at height 4, and the ray falls without end as it goes left: $(-\infty, 4]$.

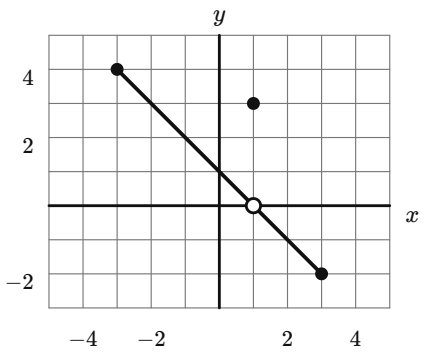
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(d) Both ends are open, at $x = -4$ and $x = 4$: $(-4, 4)$. The lowest point is the corner $(-1, -3)$ and the highest is the corner $(2, 3)$, both on the graph, and the graph is unbroken between them: $[-3, 3]$. The open ends, at heights 2 and 0, lie inside this stretch, and the graph passes those heights elsewhere, so they take nothing away.

(e) Both arrows point sideways and down, so every x is used: $(-\infty, \infty)$. The highest point is the peak at height 3, and both arms fall without end: $(-\infty, 3]$.

(f) The closed dot at $x = 3$ is the right end, and the arrow carries the curve left forever: $(-\infty, 3]$. The highest point is the dot, at height 4. Going left, the curve gets closer and closer to height -2 but never reaches it, so the range has an open end there with no open dot drawn: $(-2, 4]$.

2. The figure shows the graph of a function f: the segment from the closed dot $(-3, 4)$ to the closed dot $(3, -2)$ with the point $(1, 0)$ missing, together with the single closed dot $(1, 3)$.



(a) Is 1 in the domain of f? Explain.

(b) Is 0 in the range? Explain.

(c) Does the closed dot $(1, 3)$ add a new number to the range? Explain.

(d) Write the domain and the range in interval notation.

ANSWER

(a) Yes: the closed dot $(1, 3)$ is above $x = 1$, so $f(1) = 3$. (b) No. (c) No: the segment already reaches height 3, at $(-2, 3)$. (d) Domain $[-3, 3]$; range $[-2, 0) \cup (0, 4]$.

SOLUTION

(a) The open dot says $(1, 0)$ is not on the graph, but the closed dot $(1, 3)$ is in the same column. So the input 1 is still used, with output $f(1) = 3$, and 1 is in the domain.

(b) The segment falls steadily, so each height from -2 to 4 is reached at exactly one point of it. Its point at height 0 is the missing one, and the extra dot has height 3, not 0. No point of the graph has height 0, so 0 is not in the range.

(c) The segment falls 1 for each 1 across from $(-3, 4)$, so it passes height 3 at $(-2, 3)$, which is on the graph. The dot's height is already in the range, so it adds nothing new there. It matters only for the domain.

(d) *Domain*. The segment covers $-3 \le x \le 3$ except $x = 1$, and the dot puts 1 back: $[-3, 3]$.

Range. The segment gives every height from -2 to 4 except 0, and the dot adds nothing:

$$[-2, 0) \cup (0, 4].$$

3. The graph of a function is a single segment with one closed end and one open end. Its domain is $[-3, 5)$ and its range is $(-1, 7]$. Find the coordinates of both ends of the segment, say which end is the open dot, and say whether the segment rises or falls from left to right.

ANSWER

The closed end is $(-3, 7)$ and the open end is $(5, -1)$; the segment falls.

SOLUTION

The domain $[-3, 5)$ says the left end is at $x = -3$ and is on the graph, and the right end is at $x = 5$ and is the open dot.

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The range $(-1, 7]$ says the height 7 is reached but the height -1 is not. A segment's lowest and highest heights are at its two ends, so one end has height 7 and the other height -1. The missing height must belong to the missing point, so the open end, at $x = 5$, has height -1, and the closed end, at $x = -3$, has height 7.

The ends are $(-3, 7)$, closed, and $(5, -1)$, open. Going from left to right the height drops from 7 to -1, so the segment falls.

Check: the rising segment from $(-3, -1)$ to an open $(5, 7)$ has the same domain but range $[-1, 7)$, not $(-1, 7]$.

4. (a) A graph is the circle with center $(1, -2)$ and radius 3. It is not the graph of a function, but its shadows on the axes still give a domain and a range. Find them in interval notation.

(b) The top half of the circle, from the closed dot $(-2, -2)$ over the top to the closed dot $(4, -2)$, is the graph of a function. Find its domain and range.

ANSWER

(a) Domain $[-2, 4]$, range $[-5, 1]$. (b) Domain $[-2, 4]$, range $[-2, 1]$.

SOLUTION

(a) Every point of the circle is 3 units from the center $(1, -2)$. The leftmost point is 3 to the left of the center, $(-2, -2)$, and the rightmost is 3 to the right, $(4, -2)$, so the domain is $[-2, 4]$. The lowest point is 3 below the center, $(1, -5)$, and the highest 3 above, $(1, 1)$, so the range is $[-5, 1]$. The circle is unbroken, so every number between is reached.

(b) The top half has the same leftmost and rightmost points, both closed: domain $[-2, 4]$. Its lowest points are its two ends, at height -2, and its highest is the top of the circle, $(1, 1)$: range $[-2, 1]$. The bottom half alone would give the rest of the circle's range, $[-5, -2]$.

5. A weather station logs the air temperature T, in degrees Celsius, t hours after midnight. Its graph is made of straight pieces. The first piece runs from the closed dot $(0, -3)$ down to $(4, -6)$ and up to the open dot $(6, -2)$. The station then goes offline for repairs. It starts again at the closed dot $(9, 4)$, and the graph climbs to $(13, 11)$ and falls to the closed dot $(18, 5)$.

(a) Find the domain and range, as inequalities and in interval notation.

(b) What does the gap in the domain mean?

(c) Is 0 in the range? Elena says the air must have been 0°C at some moment between hours 6 and 9. Is she right, and does that change your answer about the range? Explain.

ANSWER

(a) Domain $0 \le t < 6$ or $9 \le t \le 18$, that is $[0, 6) \cup [9, 18]$. Range $-6 \le T < -2$ or $4 \le T \le 11$, that is $[-6, -2) \cup [4, 11]$. (b) No temperatures were logged from hour 6 until hour 9. (c) 0 is not in the range. Elena is right about the air, but the range holds only the logged temperatures, so it does not change.

SOLUTION

(a) *Domain.* The first piece covers $0 \le t < 6$ (the dot at $t = 6$ is open); the second covers $9 \le t \le 18$. Nothing lies above the hours between, so the domain is

$$[0, 6) \cup [9, 18].$$

Range. The first piece falls from -3 to its lowest point -6, then climbs toward -2 without reaching it, since that end is open and no other point of the first piece is that high: $[-6, -2)$. The second piece climbs

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from 4 to 11 and falls back to 5, so it covers $[4, 11]$. The two stretches do not overlap:

$$[-6, -2) \cup [4, 11].$$

(b) The function has no output for those inputs: the station recorded nothing from hour 6 until hour 9.

(c) 0 lies in neither stretch of the range, so no logged point has height 0: 0 is not in the range. The real air temperature changes without jumps, and it went from about −2°C to 4°C during the gap, so Elena is right that it passed 0°C. But the range is the set of outputs the graph actually has, and the graph has no points in the gap. A graph gives only what it shows, so the range stays $[-6, -2) \cup [4, 11]$.

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