Algebra 2 Unit 1 worksheet: Graphing by transforming a parent graph (answer key)

ANSWERS AT A GLANCE

1. (a) Vertex $(-3, 0)$, opens downward; domain $(-\infty, \infty)$, range $(-\infty, 0]$ (b) Asymptotes $x = 6$ and $y = 2$, branches up and to the left and down and to the right of the center; domain $(-\infty, 6) \cup (6, \infty)$, range $(-\infty, 2) \cup (2, \infty)$ (c) Endpoint $(-3, 4)$, runs right and down; domain $[-3, \infty)$, range $(-\infty, 4]$ (d) Center $\left(\frac{1}{2}, 4\right)$, falls from left to right; domain $(-\infty, \infty)$, range $(-\infty, \infty)$
2. (a) $y = \frac{1}{4}\big(-(x-2)\big)^3 + 1$: $a = \frac{1}{4}$, $b = -1$, $h = 2$, $k = 1$. (b) $(-2, -8) \mapsto (4, -1)$, $(-1, -1) \mapsto \left(3, \frac{3}{4}\right)$, $(0, 0) \mapsto (2, 1)$, $(1, 1) \mapsto \left(1, \frac{5}{4}\right)$, $(2, 8) \mapsto (0, 3)$; the sketch is a cubic through these five points. (c) It falls from left to right; center $(2, 1)$; it crosses the x-axis at $\left(2 + \sqrt[3]{4}, 0\right)$.
3. (a) $y = 2\sqrt{3(x-4)} + 1$: $a = 2$, $b = 3$, $h = 4$, $k = 1$. (b) The image of (p, q) has x-coordinate $\frac{p}{3} + 4$, a fraction unless p is a multiple of 3; for example $(9, 3)$ and $(36, 6)$. (c) $(0, 0) \mapsto (4, 1)$, $(9, 3) \mapsto (7, 7)$, $(36, 6) \mapsto (16, 13)$; domain $[4, \infty)$, range $[1, \infty)$. (d) Neither: every input is at least 4, so $x = 0$ is never reached, and every output is at least 1, so $y = 0$ is never reached.
4. (a) Exactly when $h > 0$ and $k > 0$. (b) The range is $[k, \infty)$, which always contains positive outputs, so some points of the graph have $y > 0$.
5. (a) At $x = -3$ the rule gives $\frac{2}{-8} + 4 = \frac{15}{4}$, not 5. (b) She moved the graph 5 left instead of 5 right: $x - 5$ means $h = 5$. (c) Asymptotes $x = 5$ and $y = 4$; $(1, 1) \mapsto (6, 6)$, $\left(2, \frac{1}{2}\right) \mapsto (7, 5)$, $(-1, -1) \mapsto (4, 2)$, $\left(-2, -\frac{1}{2}\right) \mapsto (3, 3)$. (d) Not the same domain (hers leaves out -5, the correct one leaves out 5); the same range, every number except 4.

WORKED SOLUTIONS

1. Each rule is a moved copy of one of the parents $y = x^2$, $y = x^3$, $y = \sqrt{x}$ or $y = \frac{1}{x}$. For each, give the new anchor (the vertex, the center, the endpoint, or both asymptotes), say which way the graph faces, and give the domain and range in interval notation.

(a) $y = -4(x+3)^2$

(b) $y = \dfrac{1}{6-x} + 2$

(c) $y = 4 - \sqrt{2x+6}$

(d) $y = -(2x-1)^3 + 4$

ANSWER

(a) Vertex $(-3, 0)$, opens downward; domain $(-\infty, \infty)$, range $(-\infty, 0]$ (b) Asymptotes $x = 6$ and $y = 2$, branches up and to the left and down and to the right of the center; domain $(-\infty, 6) \cup (6, \infty)$, range $(-\infty, 2) \cup (2, \infty)$ (c) Endpoint $(-3, 4)$, runs right and down; domain $[-3, \infty)$, range $(-\infty, 4]$ (d) Center $\left(\frac{1}{2}, 4\right)$, falls from left to right; domain $(-\infty, \infty)$, range $(-\infty, \infty)$

SOLUTION

Write each rule as $y = a\,f\big(b(x-h)\big) + k$. The anchor goes to (h, k); a negative a turns the graph upside down and a negative b turns it left for right. The domain moves with b and h, the range with a and k.

(a) $a = -4$, $b = 1$, $h = -3$, $k = 0$. Vertex $(-3, 0)$; $a < 0$, so it opens downward. A parabola takes every input; the parent's range $[0, \infty)$ becomes $(-\infty, 0]$ because the negative a swaps the ends, and $k = 0$ leaves it there.

(b) $6 - x = -(x - 6)$, so $a = 1$, $b = -1$, $h = 6$, $k = 2$. The asymptotes $x = 0$ and $y = 0$ move to $x = 6$ and $y = 2$. The negative b swaps left and right: the parent's branches sit up-right and down-left of its center, so these sit up-left and down-right of $(6, 2)$. Check: at $x = 7$, $y = \frac{1}{-1} + 2 = 1$, below 2 and to the right. The gaps at 0 move to 6 and 2.

(c) $4 - \sqrt{2x+6} = -\sqrt{2(x+3)} + 4$, so $a = -1$, $b = 2$, $h = -3$, $k = 4$. Endpoint $(-3, 4)$. Since $b > 0$ the graph still runs right, and since $a < 0$ it runs down. Domain: the end 0 goes to $\frac{0}{2} - 3 = -3$, giving

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$[-3, \infty)$. Range: $[0, \infty)$ times -1 is $(-\infty, 0]$, then up 4: $(-\infty, 4]$.

(d) $-(2x-1)^3 + 4 = -\left(2\left(x - \frac{1}{2}\right)\right)^3 + 4$, so $a = -1$, $b = 2$, $h = \frac{1}{2}$, $k = 4$. Center $\left(\frac{1}{2}, 4\right)$. The parent rises, so after the flip the graph falls from left to right. Every number is still an input and an output.

2. Graph $y = \frac{1}{4}(2 - x)^3 + 1$ by moving $y = x^3$.

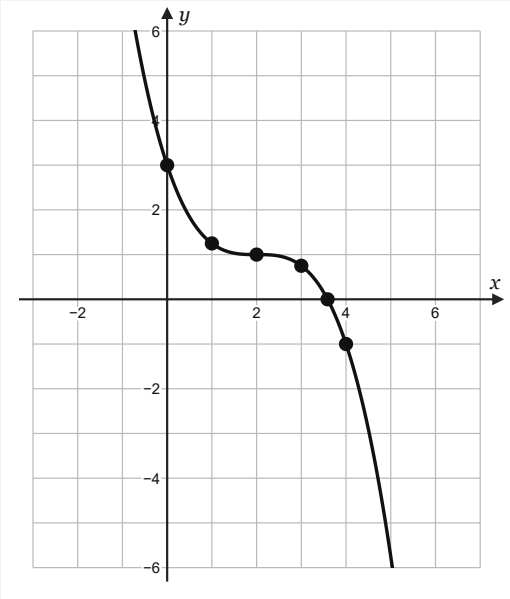
(a) Write the rule in the form $y = a\,f\big(b(x - h)\big) + k$ and give a, b, h and k.

(b) Make a table carrying the parent points (−2, −8), (−1, −1), (0, 0), (1, 1) and (2, 8), then sketch the graph through the image points.

(c) Does the graph rise or fall from left to right? Give its center, and find exactly where it crosses the x-axis.

ANSWER

(a) $y = \frac{1}{4}\big(-(x-2)\big)^3 + 1$: $a = \frac{1}{4}$, $b = -1$, $h = 2$, $k = 1$. (b) $(-2, -8) \mapsto (4, -1)$, $(-1, -1) \mapsto \left(3, \frac{3}{4}\right)$, $(0, 0) \mapsto (2, 1)$, $(1, 1) \mapsto \left(1, \frac{5}{4}\right)$, $(2, 8) \mapsto (0, 3)$; the sketch is a cubic through these five points. (c) It falls from left to right; center (2, 1); it crosses the x-axis at $\left(2 + \sqrt[3]{4}, 0\right)$.



SOLUTION

(a) Factor the inside: $2 - x = -(x - 2)$. So $y = \frac{1}{4}\big(-(x-2)\big)^3 + 1$ with $a = \frac{1}{4}$, $b = -1$, $h = 2$ and $k = 1$.

(b) Each parent point (p, q) goes to $\left(\frac{p}{-1} + 2,\ \frac{1}{4}q + 1\right) = \left(2 - p,\ \frac{q}{4} + 1\right)$.

Parent point	x: $2 - p$	y: $\frac{q}{4} + 1$	**Image**
(−2, −8)	4	−1	(4, −1)
(−1, −1)	3	$\frac{3}{4}$	$\left(3, \frac{3}{4}\right)$
(0, 0)	2	1	(2, 1)
(1, 1)	1	$\frac{5}{4}$	$\left(1, \frac{5}{4}\right)$
(2, 8)	0	3	(0, 3)

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The key for the sketch: the five image points, the center (2, 1) where the curve flattens, and a curve that comes down from the upper left and goes on down to the lower right. Check one point: $\frac{1}{4}(2-4)^3+1 = \frac{1}{4}(-8)+1 = -1$.

(c) The negative b turns the parent left for right. The parent rises from left to right, so this graph falls: the images read from left to right have heights $3, \frac{5}{4}, 1, \frac{3}{4}, -1$. The center is $(h,k)=(2,1)$. On the x-axis $y=0$:

$$\frac{1}{4}(2-x)^3+1=0 \quad\Longrightarrow\quad (2-x)^3=-4 \quad\Longrightarrow\quad 2-x=-\sqrt[3]{4} \quad\Longrightarrow\quad x=2+\sqrt[3]{4}$$

That is about 3.59, between the images at $x=3$ (height $\frac{3}{4}$) and $x=4$ (height −1), as the sketch shows.

3. Graph $y=2\sqrt{3x-12}+1$ by moving $y=\sqrt{x}$.

(a) Factor the inside and give a, b, h and k.

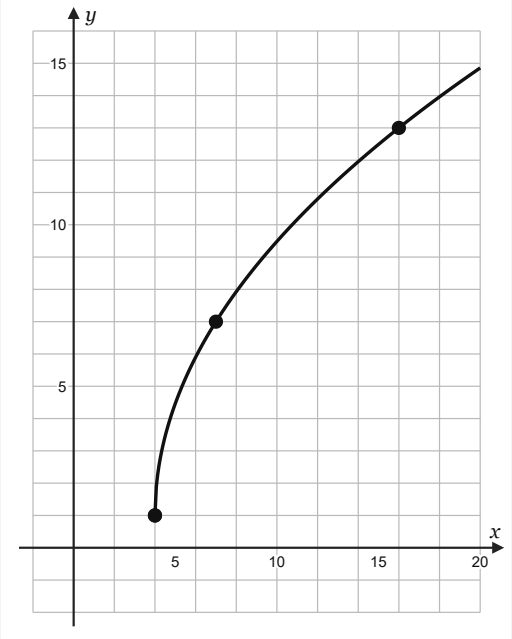
(b) Explain why the parent points (1, 1) and (4, 2) give image points with fractional x-coordinates here. Then find two parent points, other than (0, 0), whose images have whole-number coordinates.

(c) Carry (0, 0) and your two points, sketch the graph, and give its domain and range.

(d) Does the graph meet either axis? Explain using the domain and range.

ANSWER

(a) $y=2\sqrt{3(x-4)}+1$: $a=2$, $b=3$, $h=4$, $k=1$. (b) The image of (p,q) has x-coordinate $\frac{p}{3}+4$, a fraction unless p is a multiple of 3; for example (9, 3) and (36, 6). (c) $(0,0)\mapsto(4,1)$, $(9,3)\mapsto(7,7)$, $(36,6)\mapsto(16,13)$; domain $[4,\infty)$, range $[1,\infty)$. (d) Neither: every input is at least 4, so $x=0$ is never reached, and every output is at least 1, so $y=0$ is never reached.



SOLUTION

(a) $3x-12=3(x-4)$, so $y=2\sqrt{3(x-4)}+1$ with $a=2$, $b=3$, $h=4$, $k=1$.

(b) Each point (p,q) goes to $\left(\frac{p}{3}+4,\ 2q+1\right)$. For (1, 1) the new x is $\frac{1}{3}+4=\frac{13}{3}$, and for (4, 2) it is $\frac{4}{3}+4=\frac{16}{3}$: dividing by 3 leaves thirds. A parent point $(p,\sqrt{p})$ has whole-number coordinates when p is a

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perfect square, and its image then has a whole-number x when p is also a multiple of 3. The perfect squares that are multiples of 3 are 9, 36, 81, and so on, so (9, 3) and (36, 6) work (as does (81, 9)).

(c) $(0, 0) \mapsto (4, 1)$, $(9, 3) \mapsto (3 + 4,\ 6 + 1) = (7, 7)$ and $(36, 6) \mapsto (12 + 4,\ 12 + 1) = (16, 13)$. The key for the sketch: the endpoint (4, 1) as a closed dot, the curve running right and up through (7, 7) and (16, 13), rising ever more slowly. Check: $2\sqrt{3 \cdot 16 - 12} + 1 = 2 \cdot 6 + 1 = 13$.

Domain: the parent's $[0, \infty)$ has its end 0 moved to $\frac{0}{3} + 4 = 4$, giving $[4, \infty)$. Range: $[0, \infty)$ times 2 is still $[0, \infty)$, then up 1: $[1, \infty)$.

(d) A point on the y-axis has $x = 0$, but 0 is not in the domain $[4, \infty)$. A point on the x-axis has $y = 0$, but 0 is not in the range $[1, \infty)$. So the graph meets neither axis; it lies wholly in Quadrant I, starting at (4, 1).

4. Consider the graphs $y = 2\sqrt{x - h} + k$ for different numbers h and k.

(a) For which h and k does every point of the graph lie in Quadrant I, where $x > 0$ and $y > 0$, with no point on either axis?

(b) Explain why no choice of h and k puts the whole graph in Quadrant IV, where $x > 0$ and $y < 0$.

ANSWER

(a) Exactly when $h > 0$ and $k > 0$. (b) The range is $[k, \infty)$, which always contains positive outputs, so some points of the graph have $y > 0$.

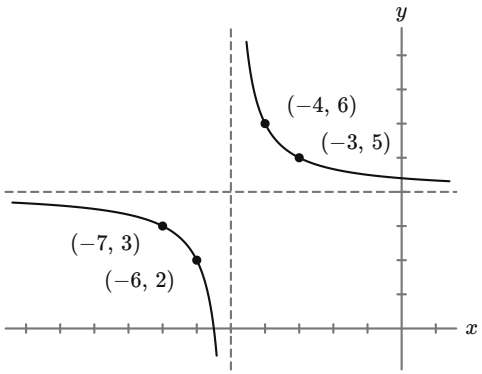
SOLUTION

Here $a = 2$ and $b = 1$. The domain of $\sqrt{x}$ is $[0, \infty)$; the shift moves it to $[h, \infty)$. The range of $\sqrt{x}$ is $[0, \infty)$; times 2 it is still $[0, \infty)$, and the shift makes it $[k, \infty)$.

(a) Every point has $x > 0$ exactly when every number in $[h, \infty)$ is positive, that is, when $h > 0$. Every point has $y > 0$ exactly when every number in $[k, \infty)$ is positive, that is, when $k > 0$. (The endpoint (h, k) is the point that comes closest to both axes, and it is in Quadrant I under these two conditions.) So the graph lies in Quadrant I exactly when $h > 0$ and $k > 0$.

(b) Whatever k is, the range $[k, \infty)$ runs upward without end, so it contains heights such as $y = |k| + 1 > 0$. The graph has points at those heights, and they lie above the x-axis, not in Quadrant IV. The domain can be made all positive, but the range can never be made all negative.

5. Jasmine was asked to graph $y = \dfrac{2}{x - 5} + 4$ by moving $y = \dfrac{1}{x}$. The figure shows her graph: she drew dashed asymptotes at $x = -5$ and $y = 4$, and carried the parent points (1, 1), $\left(2, \frac{1}{2}\right)$, (−1, −1) and $\left(-2, -\frac{1}{2}\right)$ to the four marked points.



(a) Test her marked point (−3, 5) in the rule to show that her graph is wrong.

(b) What mistake did she make?

(c) Give the correct asymptotes and the correct images of the four parent points.

(d) Do her graph and the correct graph have the same domain? The same range?

ANSWER

(a) At $x = -3$ the rule gives $\dfrac{2}{-8} + 4 = \dfrac{15}{4}$, not 5. (b) She moved the graph 5 left instead of 5 right: $x - 5$ means $h = 5$. (c) Asymptotes $x = 5$ and $y = 4$; $(1, 1) \mapsto (6, 6)$, $\left(2, \frac{1}{2}\right) \mapsto (7, 5)$, $(-1, -1) \mapsto (4, 2)$,

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$\left(-2, -\frac{1}{2}\right) \mapsto (3, 3)$. (d) Not the same domain (hers leaves out -5, the correct one leaves out 5); the same range, every number except 4.

SOLUTION

(a) $\frac{2}{-3-5} + 4 = \frac{2}{-8} + 4 = -\frac{1}{4} + 4 = \frac{15}{4}$. The point $(-3, 5)$ is not on the graph of the rule, so her graph is not the graph of the rule.

(b) Since $\frac{2}{x-5} + 4 = 2 \cdot \frac{1}{x-5} + 4$, the numbers are $a = 2$, $b = 1$, $h = 5$, $k = 4$. The minus sign in $x - 5$ moves the graph right 5. Her asymptote $x = -5$ and her points show a shift left 5: she graphed $y = \frac{2}{x+5} + 4$. Her vertical stretch and her shift up are right.

(c) The asymptotes move to $x = h = 5$ and $y = k = 4$. Each point goes to $(p + 5,\ 2q + 4)$: $(1, 1) \mapsto (6, 6)$, $\left(2, \frac{1}{2}\right) \mapsto (7, 5)$, $(-1, -1) \mapsto (4, 2)$, $\left(-2, -\frac{1}{2}\right) \mapsto (3, 3)$. Check: $\frac{2}{7-5} + 4 = 5$.

(d) The domain is every number except h: hers leaves out -5 and the correct graph leaves out 5, so the domains differ. The range is every number except k, and both graphs have $k = 4$, so the ranges are the same. Her mistake was in h, which moves only the domain.

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