Algebra 2 Unit 1 worksheet: Linear functions in context (answer key)

ANSWERS AT A GLANCE

1. (a) Input: time t in hours; output: distance left in km. Rate -80 km per hour: the distance left drops by 80 km each hour. Starting value 640 km: the whole trip. $D(5) = 240$: after 5 hours of driving, 240 km are left.
   (b) Input: mass k in kg; output: cost in dollars. Rate 0.12 dollars per kg: each kilogram adds 12 cents. Starting value 45 dollars: the fixed delivery charge in every order. $C(800) = 141$: delivering 800 kg costs 141 dollars.
   (c) Input: years y; output: salary in dollars. Rate 2400 dollars per year: her raise each year. Starting value 38000 dollars: her salary when she started. $S(4) = 47600$: 4 years after starting, her salary is 47600 dollars a year.
2. (a) $M(n) = 110n + 600$: the card's balance grows by 110 dollars per week, and it held 600 dollars at the start. (b) $M(10) = 1700$: after 10 weeks she has 1700 dollars. (c) after 12 weeks
3. (a) 50 dollars a week; $M(w) = 25w + 340$ and $K(w) = 50w + 115$. (b) 275 dollars ahead.
4. (a) $L(x) = 6x + 12$: the spring stretches 6 cm for each kilogram hung on it, and it is 12 cm long with nothing hanging. (b) 9 kg
5. (a) The price for 70 prints is wrong; it should be 17.10 dollars. (b) $C(n) = 0.18n + 4.5$: each print costs 0.18 dollars, and every order pays a fixed 4.50 dollars (for example, for shipping).
6. (a) 100 dollars per month: she puts in 100 dollars each month. (b) $B(0) = 1000$ dollars, the amount she started the fund with; the left edge of the graph is month 6, not month 0. (c) $B(m) = 100m + 1000$; 30 months after starting.
7. (a) Each shirt sold adds 7 dollars of profit; -210 dollars is the cost of the order, the loss if no shirts are sold. 30 shirts. (b) $P_1(n) = 7n - 270$: the line moves down 60, parallel to the old one; 39 shirts. (c) $P_2(n) = 10n - 210$: the line pivots about (0, -210) and is steeper; 21 shirts. (d) P_1 is 60 dollars lower for every n; P_2 is $3n$ dollars higher. At 100 shirts the price rise adds 300 dollars while the cost rise takes away only 60, so the price rise matters more.
8. For example, 12 cupcakes cost 36 dollars, but 24 cost 66 dollars, not 72. The part $2.5n$, the cost of the cupcakes themselves, doubles; the 6 dollar fixed charge is paid once per order and does not depend on n.

WORKED SOLUTIONS

1. Each rule describes a situation. For each one, name the input and the output with their units, say what the rate of change and the starting value mean in the situation (with units), and write the statement after it as a sentence.

(a) $D(t) = 640 - 80t$ is the distance, in km, a family still has to drive to reach a city t hours after leaving home. Statement: $D(5) = 240$.

(b) $C(k) = 0.12k + 45$ is the cost, in dollars, of having k kilograms of gravel delivered. Statement: $C(800) = 141$.

(c) $S(y) = 2400y + 38000$ is a nurse's yearly salary, in dollars, y years after she started her job. Statement: $S(4) = 47600$.

> **ANSWER**
> (a) Input: time t in hours; output: distance left in km. Rate -80 km per hour: the distance left drops by 80 km each hour. Starting value 640 km: the whole trip. $D(5) = 240$: after 5 hours of driving, 240 km are left.
>
> (b) Input: mass k in kg; output: cost in dollars. Rate 0.12 dollars per kg: each kilogram adds 12 cents. Starting value 45 dollars: the fixed delivery charge in every order. $C(800) = 141$: delivering 800 kg costs 141 dollars.
>
> (c) Input: years y; output: salary in dollars. Rate 2400 dollars per year: her raise each year. Starting value 38000 dollars: her salary when she started. $S(4) = 47600$: 4 years after starting, her salary is 47600 dollars a year.
>
> **SOLUTION**
> The number multiplying the input is the rate of change, in output units per input unit; the number on its own is the starting value, the output at input 0, in output units.
>
> (a) The rate -80 is negative, so the distance left goes down: they cover 80 km each hour. At $t = 0$ they have not moved, so $D(0) = 640$ km is the length of the whole trip. Check the statement: 640 − 80(5) = 640

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– 400 = 240, so after 5 hours, 240 km are left.

(b) Each kilogram adds 0.12 dollars. $C(0) = 45$ is not the price of a real delivery (nobody orders 0 kg); it is the fixed part of every price, the delivery charge. Check: 0.12(800) + 45 = 96 + 45 = 141, so 800 kg of gravel delivered costs 141 dollars.

(c) The salary rises by 2400 dollars every year, and $S(0) = 38000$ dollars is the starting salary. Check: 2400(4) + 38000 = 9600 + 38000 = 47600, so 4 years after starting she earns 47600 dollars a year.

2. Mia has 600 dollars on her bus pass card. Each week she tops it up with 120 dollars from her weekend job and spends 10 dollars of it on bus fares.

(a) Write the amount $M(n)$, in dollars, on the card after n weeks, and say what each number in your rule means, with its units.

(b) Find $M(10)$ and say what it means.

(c) After how many weeks will she have 1920 dollars?

ANSWER

(a) $M(n) = 110n + 600$: the card's balance grows by 110 dollars per week, and it held 600 dollars at the start. (b) $M(10) = 1700$: after 10 weeks she has 1700 dollars. (c) after 12 weeks

SOLUTION

(a) The input is the number of weeks n and the output is the amount in dollars. Each week brings 120 dollars in and 10 dollars out, so the card's balance grows by 120 − 10 = 110 dollars per week: that is the rate. The 600 dollars are there before any week has passed: that is the starting value.

$$M(n) = 110n + 600$$

(b) $M(10) = 110(10) + 600 = 1100 + 600 = 1700$. After 10 weeks she has 1700 dollars on the card.

(c) Solve $M(n) = 1920$: $110n + 600 = 1920$, so $110n = 1320$ and $n = 12$. She has 1920 dollars after 12 weeks.

3. Mei has 340 dollars saved and adds 25 dollars every week. Kofi has 115 dollars saved and adds the same amount every week, more than Mei does. After exactly 9 weeks they have the same amount.

(a) How much does Kofi add each week? Write each person's savings, $M(w)$ and $K(w)$ in dollars, after w weeks.

(b) How far ahead of Mei is Kofi after 20 weeks?

ANSWER

(a) 50 dollars a week; $M(w) = 25w + 340$ and $K(w) = 50w + 115$. (b) 275 dollars ahead.

SOLUTION

(a) At the start Mei is ahead by 340 − 115 = 225 dollars. Kofi closes that gap in 9 weeks, so each week he gains $\frac{225}{9} = 25$ dollars on her: he adds 25 dollars more than she does, 25 + 25 = 50 dollars a week. So $M(w) = 25w + 340$ and $K(w) = 50w + 115$.

Check: $M(9) = 225 + 340 = 565$ and $K(9) = 450 + 115 = 565$.

(b) After week 9 Kofi keeps gaining 25 dollars a week, for 20 − 9 = 11 more weeks: 11 × 25 = 275 dollars. Check: $K(20) = 1000 + 115 = 1115$ and $M(20) = 500 + 340 = 840$, and 1115 − 840 = 275.

4. A spring hangs from a hook, and its length depends on the mass hung from it. With 1 kg hanging, the spring is 18 cm long; with 4 kg hanging, it is 36 cm long. Assume the length $L(x)$, in cm, with x kg hanging is a linear function of x.

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(a) Write $L(x)$ and say what both of its numbers mean, with their units.

(b) What mass stretches the spring to 66 cm?

ANSWER

(a) $L(x) = 6x + 12$: the spring stretches 6 cm for each kilogram hung on it, and it is 12 cm long with nothing hanging. (b) 9 kg

SOLUTION

(a) Mass is the input and length the output. The pairs are (1, 18) and (4, 36), so the rate is

$$m = \frac{36 - 18}{4 - 1} = \frac{18}{3} = 6 \text{ cm per kg}$$

Walking back from 1 kg to 0 kg: $b = 18 - 6(1) = 18 - 6 = 12$. So $L(x) = 6x + 12$. Check with the other pair: 6(4) + 12 = 36.

Each kilogram stretches the spring by 6 cm, and $L(0) = 12$ cm is the spring's length with nothing on it.

(b) Solve $6x + 12 = 66$: $6x = 54$, so $x = 9$ kg.

5. An online shop charges for an order of photo prints by the number of prints, n. Its price $C(n)$, in dollars, is a linear function of n. Ryan copied these prices from the shop's website, but he copied one of them wrong.

n **(prints)**	**10**	**25**	**40**	**70**	**100**
$C(n)$ (dollars)	6.30	9.00	11.70	17.40	22.50

(a) Find the wrong price and give the right one.

(b) Write $C(n)$ and say what both of its numbers mean, with their units.

ANSWER

(a) The price for 70 prints is wrong; it should be 17.10 dollars. (b) $C(n) = 0.18n + 4.5$: each print costs 0.18 dollars, and every order pays a fixed 4.50 dollars (for example, for shipping).

SOLUTION

(a) Find the rate between neighboring rows, dividing by the step in n each time:

- 10 to 25: $\frac{9.00 - 6.30}{15} = \frac{2.70}{15} = 0.18$
- 25 to 40: $\frac{11.70 - 9.00}{15} = \frac{2.70}{15} = 0.18$
- 40 to 70: $\frac{17.40 - 11.70}{30} = \frac{5.70}{30} = 0.19$
- 70 to 100: $\frac{22.50 - 17.40}{30} = \frac{5.10}{30} = 0.17$

Only the two rates next to $n = 70$ are off, and skipping that row, 40 to 100 gives $\frac{22.50 - 11.70}{60} = \frac{10.80}{60} = 0.18$ again. So the price for 70 prints is the wrong one. From 40 prints, 30 more prints add 30(0.18) = 5.40 dollars: the right price is 11.70 + 5.40 = 17.10 dollars.

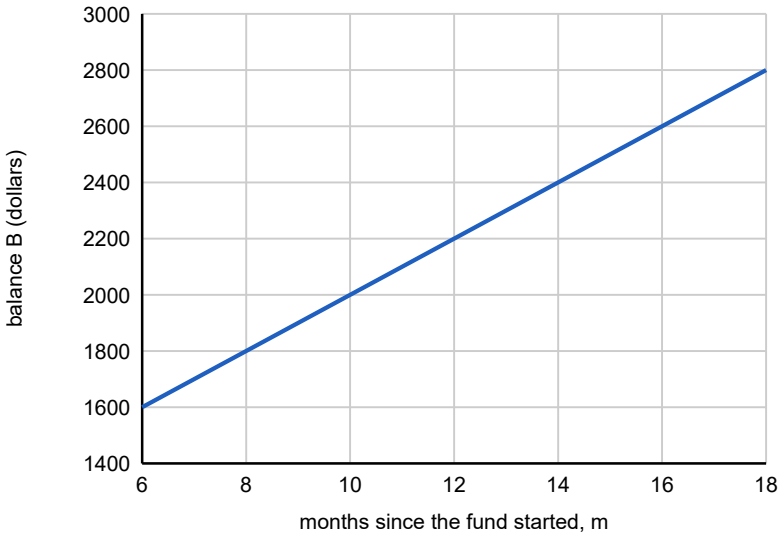
(b) The rate is 0.18 dollars per print. Walking back from 10 prints to 0: $b = 6.30 - 0.18(10) = 6.30 - 1.80 = 4.50$. So $C(n) = 0.18n + 4.5$. Check: 0.18(100) + 4.5 = 22.5.

Each print adds 0.18 dollars (18 cents). $C(0) = 4.50$ dollars is not the price of a real order of no prints; it is the fixed charge added to every order.

6. Grace is saving for a used car. She started a car fund and puts in the same amount every month. The graph shows the balance $B(m)$ of the fund, in dollars, m months after she started it, from month 6 to month 18.

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Notice where each axis starts.



(a) Find the rate of change of B, with its units, and say what it means.

(b) Find the starting value $B(0)$ and say what it means. Why is it not the number where the line meets the left edge of the graph?

(c) Write $B(m)$ and find when the balance reaches 4000 dollars, if she keeps saving the same way.

ANSWER

(a) 100 dollars per month: she puts in 100 dollars each month. (b) $B(0) = 1000$ dollars, the amount she started the fund with; the left edge of the graph is month 6, not month 0. (c) $B(m) = 100m + 1000$; 30 months after starting.

SOLUTION

(a) The line passes through the grid corners (6, 1600) and (18, 2800). Over 12 months the balance grows by 1200 dollars:

$$m = \frac{2800 - 1600}{18 - 6} = \frac{1200}{12} = 100 \text{ dollars per month}$$

She adds 100 dollars to the fund every month.

(b) The horizontal axis starts at month 6, so where the line meets the left edge is $B(6) = 1600$, not $B(0)$. Walk back the 6 months: $B(0) = 1600 - 6(100) = 1000$ dollars. That is the balance on the day she started the fund, the amount she started it with.

(c) $B(m) = 100m + 1000$. Check with the corner (10, 2000): 100(10) + 1000 = 2000. Solve $100m + 1000 = 4000$: $100m = 3000$, so $m = 30$. The balance reaches 4000 dollars 30 months after she started the fund.

7. A school club sells T-shirts. Its profit, in dollars, from selling n shirts is $P(n) = 7n - 210$.

(a) Say what 7 and −210 mean, and find the fewest shirts the club must sell to avoid a loss.

(b) The printer raises the cost of the order by 60 dollars, and nothing else changes. Write the new profit $P_1(n)$, say how its graph differs from the graph of P, and find the fewest shirts that now avoid a loss.

(c) Instead, the club raises its price by 3 dollars a shirt, and nothing else changes. Write the new profit $P_2(n)$, say how its graph differs from the graph of P, and find the fewest shirts that now avoid a loss.

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(d) For each change, by how much does the profit on n shirts differ from $P(n)$? Use this to say which change matters more for a club that sells 100 shirts.

ANSWER

(a) Each shirt sold adds 7 dollars of profit; −210 dollars is the cost of the order, the loss if no shirts are sold. 30 shirts. (b) $P_1(n) = 7n - 270$: the line moves down 60, parallel to the old one; 39 shirts. (c) $P_2(n) = 10n - 210$: the line pivots about (0, −210) and is steeper; 21 shirts. (d) P_1 is 60 dollars lower for every n; P_2 is $3n$ dollars higher. At 100 shirts the price rise adds 300 dollars while the cost rise takes away only 60, so the price rise matters more.

SOLUTION

(a) The rate 7 is in dollars per shirt: each shirt sold adds 7 dollars of profit. The starting value $P(0) = -210$ dollars says the club loses 210 dollars if it sells nothing: the cost of the order. No loss means $7n - 210 \geq 0$, so $n \geq 30$: at least 30 shirts.

(b) The starting value drops by 60 and the rate stays: $P_1(n) = 7n - 270$. Every output is 60 dollars lower, so the graph moves straight down 60 and stays parallel. No loss means $7n \geq 270$, $n \geq 38.57\ldots$, and shirts come in whole numbers, so at least 39 shirts.

(c) The rate rises by 3 and the starting value stays: $P_2(n) = 10n - 210$. The point (0, −210) stays put and the line turns about it, getting steeper. No loss means $10n \geq 210$, so at least 21 shirts.

(d) $P_1(n) - P(n) = (7n - 270) - (7n - 210) = -60$, the same for every n. $P_2(n) - P(n) = (10n - 210) - (7n - 210) = 3n$, which grows with n. At 100 shirts: $P(100) = 490$, $P_1(100) = 430$ (down 60) and $P_2(100) = 790$ (up 300). For a big order, the price change matters far more.

8. A bakery charges $C(n) = 2.5n + 6$ dollars for an order of n cupcakes. Liam says: "If I order twice as many cupcakes, I pay twice as much." Show with an example that Liam is wrong. Then say which part of the price does double, and why the rest does not.

ANSWER

For example, 12 cupcakes cost 36 dollars, but 24 cost 66 dollars, not 72. The part $2.5n$, the cost of the cupcakes themselves, doubles; the 6 dollar fixed charge is paid once per order and does not depend on n.

SOLUTION

Take $n = 12$: $C(12) = 2.5(12) + 6 = 30 + 6 = 36$ dollars. Twice as many: $C(24) = 2.5(24) + 6 = 60 + 6 = 66$ dollars. Twice 36 is 72, so doubling the order did not double the price.

Doubling n doubles $2.5n$, the 2.5 dollars per cupcake times the number of cupcakes: from 30 to 60. The starting value 6 is a fixed charge for the order, the same whatever n is, so it is paid once and not doubled. The price of the bigger order is 6 dollars less than double.

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