Algebra 2 Unit 1 worksheet: Symmetry about the vertical axis and the origin (answer key)

ANSWERS AT A GLANCE

1. p: about the origin; $p(-2) = 4$, $p(0) = 0$, $p(3) = -7$.
   q: about the vertical axis; $q(-3) = -5$, $q(-1) = 1$.
   r: the table cannot decide; $r(-2) = 0$ either way ($r(-3)$ is 5 or −5, and $r(1)$ is 8 or −8).
2. (a) $f(10) = 98$ and $f(-10) = 94$: the point $(10, 98)$ has no mirror partner, so the graph is not symmetric about the vertical axis.
   (b) $f(3) - f(-3) = \dfrac{27}{250}$, about 0.1 of a unit, too small to see on this grid.
   (c) No: $f(0) = -4$, so the graph does not pass through the origin.
3. (a) keeps (b) breaks (c) keeps (d) keeps (e) breaks (f) keeps
4. (a) $f(-4) = -3$ and $f(-3) = -4$.
   (b) $x = -7$ and $x = 7$.
   (c) $-5 < x < -1$ or $1 < x < 5$.
   (d) Lowest points $(-3, -4)$ and $(3, -4)$; the graph rises for $-3 < x < 0$ and for $x > 3$.

WORKED SOLUTIONS

1. Each table is for a function whose graph is symmetric about the vertical axis or about the origin. For each table, decide which symmetry it has, naming the entries that decide it, and fill in every missing entry the symmetry forces. If the entries shown cannot decide, say so, and fill in only the entries that come out the same either way.

x	−3	−2	−1	0	1	2	3
$p(x)$	7	?	−2	?	2	−4	?
$q(x)$	?	6	?	3	1	6	−5
$r(x)$	?	?	8	0	?	0	5

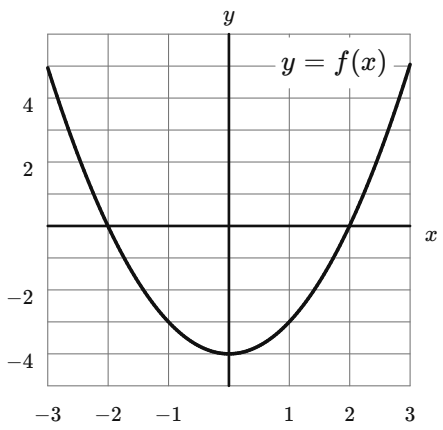
   ANSWER
   p: about the origin; $p(-2) = 4$, $p(0) = 0$, $p(3) = -7$.
   q: about the vertical axis; $q(-3) = -5$, $q(-1) = 1$.
   r: the table cannot decide; $r(-2) = 0$ either way ($r(-3)$ is 5 or −5, and $r(1)$ is 8 or −8).

   SOLUTION
   p: the entries $p(-1) = -2$ and $p(1) = 2$ are opposites, not equal, so the graph is not symmetric about the vertical axis; it must be symmetric about the origin. Each (a, b) has the partner $(-a, -b)$: $p(-2) = -p(2) = 4$ and $p(3) = -p(-3) = -7$. A function's graph symmetric about the origin with a point above $x = 0$ passes through the origin, so $p(0) = 0$.
   q: $q(0) = 3$, not 0, so the graph does not pass through the origin and cannot be symmetric about it; it is symmetric about the vertical axis. (The pair $q(-2) = q(2) = 6$ agrees.) Each (a, b) has the partner $(-a, b)$: $q(-3) = q(3) = -5$ and $q(-1) = q(1) = 1$.
   r: no pair of opposite inputs has both entries shown except 0, and $r(0) = 0$ fits both symmetries. So the table cannot decide. $r(-2)$ is $r(2) = 0$ for the vertical axis and $-r(2) = 0$ for the origin, so $r(-2) = 0$ either way. But $r(-3)$ is 5 or −5, and $r(1)$ is 8 or −8, depending on which symmetry it is.

2. Mia graphs $f(x) = \dfrac{x^3}{500} + x^2 - 4$ on the window $-3 \le x \le 3$ shown. She says the graph is symmetric about the vertical axis, because the left and right halves look like mirror images.
   (a) Find $f(10)$ and $f(-10)$. What do they show about Mia's claim?
   (b) Find $f(3) - f(-3)$ exactly, and explain why the figure does not show the difference.

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(c) Is the graph symmetric about the origin? Use one point to decide.



ANSWER

(a) $f(10) = 98$ and $f(-10) = 94$: the point (10, 98) has no mirror partner, so the graph is not symmetric about the vertical axis.

(b) $f(3) - f(-3) = \dfrac{27}{250}$, about 0.1 of a unit, too small to see on this grid.

(c) No: $f(0) = -4$, so the graph does not pass through the origin.

SOLUTION

(a) $f(10) = \dfrac{1000}{500} + 100 - 4 = 98$ and $f(-10) = -\dfrac{1000}{500} + 100 - 4 = 94$. If the graph were symmetric about the vertical axis, the partner of (10, 98) would be (−10, 98), but the height above $x = -10$ is 94. One point with no partner rules the symmetry out. Mia's picture only shows a window, and matching halves inside it make the symmetry believable, not certain.

(b) $f(3) = \dfrac{27}{500} + 9 - 4$ and $f(-3) = -\dfrac{27}{500} + 9 - 4$, so $f(3) - f(-3) = \dfrac{54}{500} = \dfrac{27}{250} = 0.108$. On a grid with squares one unit tall, a tenth of a square is about the width of the curve's line, so the two heights look equal. Farther out the $\dfrac{x^3}{500}$ term grows and the halves pull apart.

(c) $f(0) = 0 + 0 - 4 = -4$, so (0, −4) is on the graph. A function's graph symmetric about the origin must pass through the origin, so it is not symmetric about the origin.

3. The functions f and h each have a point above every x. The graph of f is symmetric about the vertical axis, and the graph of h is symmetric about the origin. Some of their values:

x	−2	−1	0	1	2
$f(x)$	5	0	−3	0	5
$h(x)$	−6	2	0	−2	6

For each new function, decide whether its graph keeps the symmetry of the function it is built from (the vertical axis for one built from f, the origin for one built from h). If it keeps it, explain why each partner pair is still a partner pair and give one pair from the table. If it breaks it, give a point on the new graph whose partner is missing.

(a) $y = f(x) + 4$

(b) $y = f(x - 1)$

(c) $y = -2f(x)$

(d) $y = f(3x)$

(e) $y = h(x) + 4$

(f) $y = -2h(x)$

ANSWER

(a) keeps (b) breaks (c) keeps (d) keeps (e) breaks (f) keeps

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SOLUTION

(a) Keeps. Partners (a, b) and $(-a, b)$ both move up 4, to $(a, b+4)$ and $(-a, b+4)$: still the same height on opposite sides. From the table: (2, 9) and (−2, 9).

(b) Breaks. The graph moves right 1, so the point (0, −3) of f goes to (1, −3). Its partner would be (−1, −3), but the new function at $x = -1$ is $f(-2) = 5$. The mirror line has moved to $x = 1$.

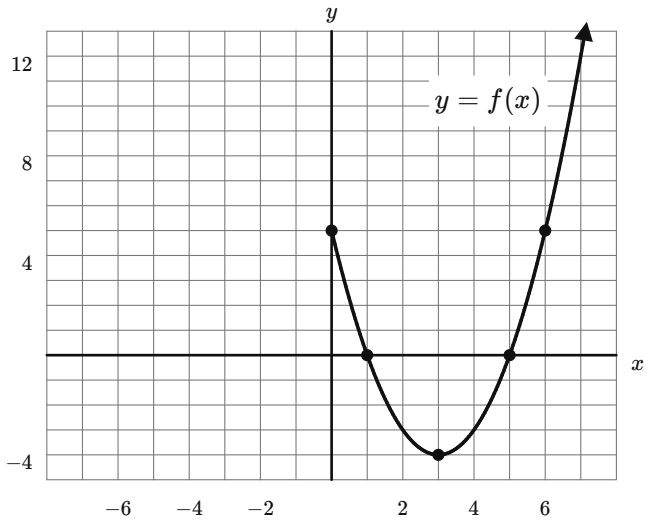
(c) Keeps. (a, b) and $(-a, b)$ go to $(a, -2b)$ and $(-a, -2b)$: equal heights again. From the table: (2, −10) and (−2, −10).

(d) Keeps. The outputs at $x = a$ and $x = -a$ are $f(3a)$ and $f(-3a)$, and $3a$ and $-3a$ are opposite inputs of f, so the two outputs are equal. From the table: $\left(\frac{2}{3}, 5\right)$ and $\left(-\frac{2}{3}, 5\right)$.

(e) Breaks. The new graph passes through (0, 4), and a function's graph symmetric about the origin must pass through the origin. (Also (1, 2) is on it, but at $x = -1$ the height is 2 + 4 = 6, not −2.)

(f) Keeps. (a, b) and $(-a, -b)$ go to $(a, -2b)$ and $(-a, 2b)$, and these are again opposite points through the origin. From the table: (1, 4) and (−1, −4).

4. The figure shows only the part of the graph of a function f for $x \geq 0$, where $f(x) = x^2 - 6x + 5$. The whole graph has a point above every x and is symmetric about the vertical axis.



(a) Find $f(-4)$ and $f(-3)$.

(b) Find every x with $f(x) = 12$.

(c) Find every x with $f(x) < 0$.

(d) Give the lowest points of the whole graph, and every stretch of x on which the whole graph rises.

ANSWER

(a) $f(-4) = -3$ and $f(-3) = -4$.

(b) $x = -7$ and $x = 7$.

(c) $-5 < x < -1$ or $1 < x < 5$.

(d) Lowest points (−3, −4) and (3, −4); the graph rises for $-3 < x < 0$ and for $x > 3$.

SOLUTION

(a) Each point (a, b) has the partner $(-a, b)$, so $f(-4) = f(4) = 16 - 24 + 5 = -3$ and $f(-3) = f(3) = 9 - 18 + 5 = -4$.

(b) For $x \geq 0$: $x^2 - 6x + 5 = 12$ gives $x^2 - 6x - 7 = 0$, $(x - 7)(x + 1) = 0$, and only $x = 7$ has $x \geq 0$. (The rule does not hold at $x = -1$; there $f(-1) = f(1) = 0$.) The partner of (7, 12) is (−7, 12), so $x = -7$

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and $x = 7$.

(c) For $x \geq 0$, $f(x) = (x - 1)(x - 5)$ is negative between its zeros, for $1 < x < 5$. Heights are kept by the fold, so the mirror stretch $-5 < x < -1$ is below the axis too.

(d) The right part is lowest at its vertex $(3, -4)$, and its mirror partner $(-3, -4)$ is a low point of the same kind. The right part falls from $x = 0$ to $x = 3$ and rises for $x > 3$. A fold turns rising into falling on the mirror stretch, so the left part falls for $x < -3$ and rises from $x = -3$ to $x = 0$. The whole graph rises for $-3 < x < 0$ and for $x > 3$, with a peak at $(0, 5)$ between.

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