Algebra 2 Unit 1 worksheet: Transforming the absolute value graph (answer key)

ANSWERS AT A GLANCE

1. (a) $y = 2|x - 3|$; axis $x = 3$; y-intercept $(0, 6)$ (b) $y = -2|x + 2| + 3$; axis $x = -2$; y-intercept $(0, -1)$ (c) $y = 3|x + 3| - 1$; axis $x = -3$; y-intercept $(0, 8)$ (d) $y = -|x - 4| + 5$; axis $x = 4$; y-intercept $(0, 1)$
2. (a) Vertex $(3, -1)$; $y = 2|x - 3| - 1$. (b) Right arm: $y = -\frac{1}{2}x + 4$ for $x \geq -4$. Left arm: $y = \frac{1}{2}x + 8$ for $x \leq -4$.
3. (a) $y = 2|x + 3| + 1$, vertex $(-3, 1)$. (b) $y = -2|x - 3| - 1$, vertex $(3, -1)$, range $y \leq -1$.
4. (a) $(1, 3)$ (b) exactly when $a < 0$ (c) $a = -1$
5. (a) $y = \frac{5}{2}|x - 2| - 3$ (b) $\left(\frac{4}{5}, 0\right)$ and $\left(\frac{16}{5}, 0\right)$
6. Vertex $(2, -3)$; endpoints $(-2, 3)$ and $(4, 0)$; intercepts $(0, 0)$ and $(4, 0)$. Range $-3 \leq y \leq 3$.

WORKED SOLUTIONS

1. Write each rule in the form $y = a|x - h| + k$. Then give the axis of symmetry of its graph (the vertical line the V is symmetric about) and the y-intercept.

(a) $y = |6 - 2x|$ (b) $y = -\frac{1}{2}|4x + 8| + 3$ (c) $y = |-3x - 9| - 1$ (d) $y = 5 - \frac{1}{3}|3x - 12|$

ANSWER

(a) $y = 2|x - 3|$; axis $x = 3$; y-intercept $(0, 6)$ (b) $y = -2|x + 2| + 3$; axis $x = -2$; y-intercept $(0, -1)$ (c) $y = 3|x + 3| - 1$; axis $x = -3$; y-intercept $(0, 8)$ (d) $y = -|x - 4| + 5$; axis $x = 4$; y-intercept $(0, 1)$

SOLUTION

In each rule, take the factor out of the bars with $|mn| = |m|\,|n|$, so that x stands alone inside. The axis of symmetry is the vertical line $x = h$ through the vertex.

(a) $6 - 2x = -2(x - 3)$ and $|-2| = 2$, so $y = 2|x - 3|$. Axis $x = 3$. At $x = 0$: $|6| = 6$, so the y-intercept is $(0, 6)$.

(b) $4x + 8 = 4(x + 2)$, so $-\frac{1}{2}|4x + 8| = -\frac{1}{2} \cdot 4|x + 2| = -2|x + 2|$ and $y = -2|x + 2| + 3$. Axis $x = -2$. At $x = 0$: $-\frac{1}{2} \cdot 8 + 3 = -1$, so $(0, -1)$.

(c) $-3x - 9 = -3(x + 3)$ and $|-3| = 3$, so $y = 3|x + 3| - 1$. A minus sign inside the bars does not flip the V. Axis $x = -3$. At $x = 0$: $|-9| - 1 = 8$, so $(0, 8)$.

(d) $3x - 12 = 3(x - 4)$, so $\frac{1}{3}|3x - 12| = |x - 4|$, and reordering the sum gives $y = -|x - 4| + 5$. Axis $x = 4$. At $x = 0$: $5 - \frac{1}{3} \cdot 12 = 1$, so $(0, 1)$.

2. Each arm of a V is a ray, part of a line.

(a) The right arm of a V lies on the line $y = 2x - 7$ and its left arm on the line $y = -2x + 5$. Find the vertex and write the rule of the V in the form $y = a|x - h| + k$.

(b) Write, in slope-intercept form, the equations of the two lines that carry the arms of $y = -\frac{1}{2}|x + 4| + 6$, and say for which inputs each arm is drawn.

ANSWER

(a) Vertex $(3, -1)$; $y = 2|x - 3| - 1$. (b) Right arm: $y = -\frac{1}{2}x + 4$ for $x \geq -4$. Left arm: $y = \frac{1}{2}x + 8$ for $x \leq -4$.

SOLUTION

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(a) The vertex is where the two arms meet, so it is on both lines: $2x - 7 = -2x + 5$ gives $4x = 12$, $x = 3$, and $y = 2(3) - 7 = -1$. The vertex is $(3, -1)$. The right arm has slope a, so $a = 2$, and the left arm's slope -2 agrees with $-a$. The rule is $y = 2|x - 3| - 1$.

Check: for $x \geq 3$, $2(x - 3) - 1 = 2x - 7$; for $x < 3$, $-2(x - 3) - 1 = -2x + 5$.

(b) Here $a = -\frac{1}{2}$, $h = -4$ and $k = 6$. To the right of the vertex, $|x + 4| = x + 4$, so $y = -\frac{1}{2}(x + 4) + 6 = -\frac{1}{2}x + 4$, drawn for $x \geq -4$. To the left, $|x + 4| = -(x + 4)$, so $y = \frac{1}{2}(x + 4) + 6 = \frac{1}{2}x + 8$, drawn for $x \leq -4$. Both give $y = 6$ at $x = -4$, the vertex $(-4, 6)$.

3. Let $f(x) = 2|x - 3| + 1$.

(a) Reflect the graph of f across the y-axis. Write the new rule in the form $y = a|x - h| + k$ and give its vertex.

(b) Reflect the graph of f across the x-axis instead. Write the new rule and give its vertex and range.

ANSWER

(a) $y = 2|x + 3| + 1$, vertex $(-3, 1)$. (b) $y = -2|x - 3| - 1$, vertex $(3, -1)$, range $y \leq -1$.

SOLUTION

(a) Reflecting across the y-axis replaces x with $-x$: $y = 2|-x - 3| + 1$. Since $-x - 3 = -(x + 3)$ and $|-1| = 1$, this is $y = 2|x + 3| + 1$. The vertex $(3, 1)$ moves to its mirror image $(-3, 1)$.

(b) Reflecting across the x-axis makes every output its opposite: $y = -(2|x - 3| + 1) = -2|x - 3| - 1$. The vertex is $(3, -1)$; now $a = -2 < 0$, so the vertex is the highest point and the range is $y \leq -1$.

4. Think about every graph $y = a|x - 1| + 3$, where a can be any number other than 0.

(a) Name the one point that is on every one of these graphs.

(b) For which values of a does the graph cross the x-axis? Explain.

(c) Find the value of a for which the two x-intercepts are 6 units apart.

ANSWER

(a) $(1, 3)$ (b) exactly when $a < 0$ (c) $a = -1$

SOLUTION

(a) At $x = 1$ the bars are 0, so $y = 3$ whatever a is. Every graph has its vertex at $(1, 3)$. (At any other x the height $a|x - 1| + 3$ changes with a.)

(b) If $a > 0$ the vertex $(1, 3)$ is the lowest point, so every height is at least 3 and the graph never reaches the x-axis. If $a < 0$ the vertex is the highest point, the arms fall forever, and each arm crosses the x-axis once. So the graph crosses exactly when $a < 0$.

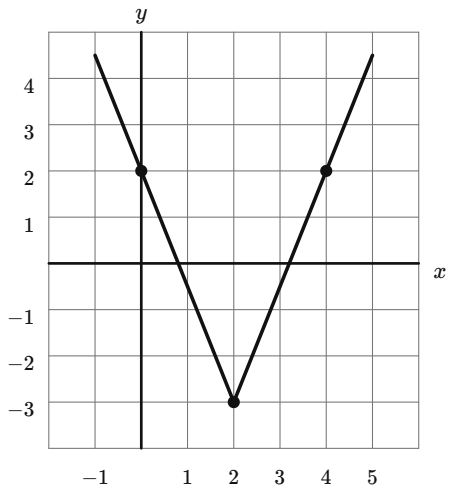
(c) The intercepts sit at the same distance from the axis $x = 1$, one on each side, so each is 3 units from 1: $x = -2$ and $x = 4$. The point $(4, 0)$ is on the V, so $a = \dfrac{0 - 3}{|4 - 1|} = -1$. Check: $-|-2 - 1| + 3 = 0$.

5. The figure shows part of the graph of a rule of the form $y = a|x - h| + k$. The three marked points sit on grid points.

(a) Write the rule.

(b) Find the x-intercepts exactly.

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ANSWER

(a) $y = \frac{5}{2}|x - 2| - 3$ (b) $\left(\frac{4}{5}, 0\right)$ and $\left(\frac{16}{5}, 0\right)$

SOLUTION

(a) The corner is at (2, −3), so $h = 2$ and $k = -3$. The marked point (0, 2) is on the V, so

$$a = \frac{y - k}{|x - h|} = \frac{2 - (-3)}{|0 - 2|} = \frac{5}{2}$$

The rule is $y = \frac{5}{2}|x - 2| - 3$. Check with (4, 2): $\frac{5}{2} \cdot 2 - 3 = 2$.

(b) The crossings do not land on grid points, so find them by algebra: $\frac{5}{2}|x - 2| - 3 = 0$ gives $|x - 2| = \frac{6}{5}$, so $x = 2 - \frac{6}{5} = \frac{4}{5}$ and $x = 2 + \frac{6}{5} = \frac{16}{5}$. The x-intercepts are $\left(\frac{4}{5}, 0\right)$ and $\left(\frac{16}{5}, 0\right)$, the same distance from the axis $x = 2$ on each side.

6. On graph paper, sketch $y = \frac{3}{2}|x - 2| - 3$ for $-2 \le x \le 4$ only, with a closed dot at each end. Mark the vertex, both endpoints and every intercept. Then give the range of this piece of the graph, and explain why it is not $y \ge -3$.

ANSWER

Vertex (2, −3); endpoints (−2, 3) and (4, 0); intercepts (0, 0) and (4, 0). Range $-3 \le y \le 3$.

SOLUTION

Here $a = \frac{3}{2}$, $h = 2$ and $k = -3$, so the vertex is (2, −3), the lowest point, and the arms have slopes $-\frac{3}{2}$ (left) and $\frac{3}{2}$ (right).

Endpoints: at $x = -2$, $y = \frac{3}{2} \cdot 4 - 3 = 3$, so (−2, 3); at $x = 4$, $y = \frac{3}{2} \cdot 2 - 3 = 0$, so (4, 0). At $x = 0$, $y = \frac{3}{2} \cdot 2 - 3 = 0$, so the graph passes through the origin. Setting $y = 0$ gives $|x - 2| = 2$, so $x = 0$ or $x = 4$: the x-intercepts are (0, 0) and (4, 0), and (0, 0) is also the y-intercept.

The sketch is the left arm from (−2, 3) down through (0, 0) to (2, −3), then the right arm up to (4, 0), closed dots at both ends.

The lowest height is −3 at the vertex. The heights grow with distance from $x = 2$, and the left end is 4 units away while the right end is only 2, so the highest is 3, at $x = -2$. The range is $-3 \le y \le 3$. It is not

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> $y \geq -3$ because the inputs stop at -2 and 4, so the arms never climb above 3.

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