Algebra 2 Unit 1 worksheet: Shifting a graph up, down, left and right (answer key)

ANSWERS AT A GLANCE

1. (a) $y = x^3$ moved 4 down; center $(0, -4)$ (b) $y = \dfrac{1}{x}$ moved 3 right and 2 down; asymptotes cross at $(3, -2)$ (c) $y = x^3$ moved 2.5 left; center $(-2.5, 0)$ (d) $y = \sqrt{x}$ moved 1 right and 6 up; starting point $(1, 6)$
2. (a) $(-6, -4)$, $(-4, 0)$, $(-1, 0)$ and $(1, -3)$ (b) Domain $-6 \le x \le 1$, range $-4 \le y \le 0$ (c) $g(0) = -1.5$
3. (a) x-intercepts $(-7, 0)$ and $(-1, 0)$; y-intercept $(0, 7)$ (b) x-intercept $(-5, 0)$; y-intercept $(0, 1)$ (c) x-intercept $(3, 0)$; y-intercept $(0, -9)$ (d) no x-intercept, since the vertex $(3, 2)$ is above the x-axis and the parabola opens up; y-intercept $(0, 11)$
4. $g(x) = x^2 - 16x + 63$; the point $(0, 0)$ lands on $(6, 3)$ and $g(6) = 3$.
5. (a) $g(x) = f(x + 5) - 2$; highest point $(1, 6)$ (b) A translation keeps the length of the domain, which is 14 for f, but $0 \le x \le 16$ has length 16.
6. (a) $h = 2$ or $h = 8$ (b) $k = -3$ (c) $h = 7$ only
7. (a) $T(t) = D(t - 1) + 0.5$ (b) $t = 3$, 5 and 7 (3 a.m., 5 a.m. and 7 a.m.) (c) $D(t + 1) + 0.5$ moves the graph 1 to the left, so every depth would come 1 hour earlier, not later.

WORKED SOLUTIONS

1. For each graph, name the parent graph, describe how it is moved, and give the point where the parent's anchor lands (the vertex, the center, the starting point, or the point where the asymptotes cross).

(a) $y = x^3 - 4$

(b) $y = \dfrac{1}{x - 3} - 2$

(c) $y = (x + 2.5)^3$

(d) $y = 6 + \sqrt{x - 1}$

ANSWER

(a) $y = x^3$ moved 4 down; center $(0, -4)$ (b) $y = \dfrac{1}{x}$ moved 3 right and 2 down; asymptotes cross at $(3, -2)$ (c) $y = x^3$ moved 2.5 left; center $(-2.5, 0)$ (d) $y = \sqrt{x}$ moved 1 right and 6 up; starting point $(1, 6)$

SOLUTION

Write each rule as $f(x - h) + k$. The anchor $(0, 0)$ moves to (h, k).

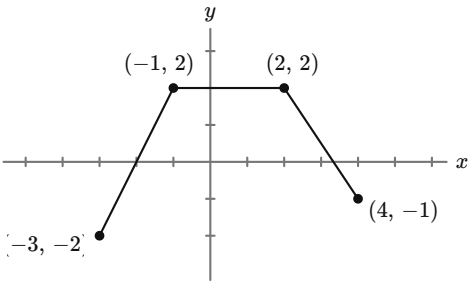
(a) $x^3 - 4 = f(x) + (-4)$ with $f(x) = x^3$: $h = 0$, $k = -4$. Down 4; the center goes to $(0, -4)$.

(b) $\dfrac{1}{x - 3} - 2 = f(x - 3) + (-2)$ with $f(x) = \dfrac{1}{x}$: right 3, down 2. The asymptotes become $x = 3$ and $y = -2$, crossing at $(3, -2)$.

(c) $(x + 2.5)^3 = f(x - (-2.5))$: $h = -2.5$. Left 2.5; center $(-2.5, 0)$.

(d) $6 + \sqrt{x - 1} = f(x - 1) + 6$: the order of the terms does not matter. Right 1, up 6; starting point $(1, 6)$.

2. The figure shows the whole graph of a function f: three segments joining the marked points, with closed dots at both ends. Let $g(x) = f(x + 3) - 2$.



(a) Where do the four marked points land on the graph of g?

(b) Give the domain and the range of g.

(c) Find $g(0)$.

ANSWER

(a) $(-6, -4)$, $(-4, 0)$, $(-1, 0)$ and $(1, -3)$ (b) Domain $-6 \le x \le 1$, range $-4 \le y \le 0$ (c) $g(0) = -1.5$

SOLUTION

Free sample from math.wiki for classroom use. Do not post. math.wiki

$g(x) = f(x-(-3)) + (-2)$, so $h = -3$ and $k = -2$: every point (a, b) moves 3 left and 2 down, to $(a-3, b-2)$.

(a) $(-3, -2) \to (-6, -4)$, $(-1, 2) \to (-4, 0)$, $(2, 2) \to (-1, 0)$ and $(4, -1) \to (1, -3)$.

(b) The graph of f runs from $x = -3$ to $x = 4$, so its domain is $-3 \le x \le 4$; its lowest point is (−3, −2) and its highest points are on the level piece at height 2, so its range is $-2 \le y \le 2$. Moving 3 left shifts the domain to $-6 \le x \le 1$; moving 2 down shifts the range to $-4 \le y \le 0$.

(c) $g(0) = f(3) - 2$. The input 3 lies on the segment from (2, 2) to (4, −1), whose slope is $\dfrac{-1-2}{4-2} = -\dfrac{3}{2}$, so $f(3) = 2 - \dfrac{3}{2} = \dfrac{1}{2}$. Then $g(0) = \dfrac{1}{2} - 2 = -1.5$. Check on the moved graph: (0, −1.5) lies on the segment from (−1, 0) to (1, −3), halfway between their heights.

3. Find the x-intercepts and the y-intercept of each graph. Give exact values, or explain why a graph has no x-intercept.

(a) $y = (x+4)^2 - 9$

(b) $y = \sqrt{x+9} - 2$

(c) $y = (x-1)^3 - 8$

(d) $y = (x-3)^2 + 2$

ANSWER

(a) x-intercepts (−7, 0) and (−1, 0); y-intercept (0, 7) (b) x-intercept (−5, 0); y-intercept (0, 1) (c) x-intercept (3, 0); y-intercept (0, −9) (d) no x-intercept, since the vertex (3, 2) is above the x-axis and the parabola opens up; y-intercept (0, 11)

SOLUTION

An x-intercept is where $y = 0$; the y-intercept is the output at $x = 0$.

(a) $(x+4)^2 = 9$, so $x + 4 = 3$ or $x + 4 = -3$: $x = -1$ or $x = -7$. At $x = 0$: 16 − 9 = 7.

(b) $\sqrt{x+9} = 2$, so $x + 9 = 4$ and $x = -5$. At $x = 0$: $\sqrt{9} - 2 = 1$.

(c) $(x-1)^3 = 8$, so $x - 1 = 2$ and $x = 3$ (a cube has only one real cube root). At $x = 0$: $(-1)^3 - 8 = -9$.

(d) $(x-3)^2 = -2$ has no real solution, because a square is never negative. On the graph: $y = x^2$ moved 3 right and 2 up has its lowest point at (3, 2), so every output is at least 2 and the graph never reaches the x-axis. At $x = 0$: 9 + 2 = 11.

4. Let $f(x) = x^2 - 4x$. Move the graph of f 6 units right and 3 units up to get the graph of g. Write $g(x)$ in the form $x^2 + bx + c$. Then check your formula: (0, 0) is on the graph of f, so find where it lands and test that point in your formula.

ANSWER

$g(x) = x^2 - 16x + 63$; the point (0, 0) lands on (6, 3) and $g(6) = 3$.

SOLUTION

Moving 6 right and 3 up gives $g(x) = f(x-6) + 3$. Replace every x in f by $x - 6$:

$$g(x) = (x-6)^2 - 4(x-6) + 3$$

Expand each piece: $(x-6)^2 = x^2 - 12x + 36$ and $-4(x-6) = -4x + 24$. Collect like terms:

$$g(x) = x^2 - 16x + 63$$

Check: the point (0, 0) moves 6 right and 3 up to (6, 3). The formula gives $g(6) = 36 - 96 + 63 = 3$, as it should.

5. A function f has domain $-7 \le x \le 7$ and range $-1 \le y \le 8$, and the highest point of its graph is (6, 8).

Free sample from math.wiki for classroom use. Do not post. math.wiki

(a) The graph of g is a translation of the graph of f, with domain $-12 \le x \le 2$ and range $-3 \le y \le 6$. Write $g(x)$ in terms of f, and give the highest point of the graph of g.

(b) Explain why no translation of the graph of f has domain $0 \le x \le 16$.

ANSWER

(a) $g(x) = f(x+5) - 2$; highest point $(1, 6)$ (b) A translation keeps the length of the domain, which is 14 for f, but $0 \le x \le 16$ has length 16.

SOLUTION

(a) A horizontal shift moves the domain and leaves the range alone; a vertical shift moves the range and leaves the domain alone. The domain moved from −7 to −12 at its left end (and from 7 to 2 at its right end), so $h = -12 - (-7) = -5$. The range moved from −1 to −3 at its bottom (and from 8 to 6 at its top), so $k = -3 - (-1) = -2$. Putting these into $g(x) = f(x-h) + k$ gives

$$g(x) = f(x+5) - 2$$

Every point (x, y) goes to $(x-5, y-2)$, so the highest point $(6, 8)$ goes to $(1, 6)$.

(b) A translation adds the same h to both ends of the domain, so the domain keeps its length, $7 - (-7) = 14$. The interval $0 \le x \le 16$ has length 16, so it cannot be the domain of any translation of f.

6. (a) The graph of $y = (x-h)^2$ passes through $(5, 9)$. Find every possible value of h, and explain why there is more than one.

(b) The graph of $y = \sqrt{x} + k$ passes through $(16, 1)$. Find k.

(c) The graph of $y = (x-h)^3$ passes through $(5, -8)$. Find every possible value of h, and explain why this time there is only one.

ANSWER

(a) $h = 2$ or $h = 8$ (b) $k = -3$ (c) $h = 7$ only

SOLUTION

(a) Put in the point: $(5-h)^2 = 9$, so $5 - h = 3$ or $5 - h = -3$, giving $h = 2$ or $h = 8$. The parabola reaches height 9 at two inputs, 3 either side of its vertex. Moving it 2 right puts its right-hand point of height 9 at $x = 5$; moving it 8 right puts its left-hand one there.

(b) $\sqrt{16} + k = 1$, so $4 + k = 1$ and $k = -3$: the curve is $y = \sqrt{x}$ moved 3 down. A vertical shift is pinned down by one point, because the input 16 fixes the parent's output 4.

(c) $(5-h)^3 = -8$, so $5 - h = -2$ and $h = 7$. The cubing graph rises the whole way and reaches each height only once, so only one shift puts the height −8 at $x = 5$.

7. The water depth at a fishing dock t hours after midnight on Monday is $D(t)$ meters:

t	0	2	4	6	8	10	12
$D(t)$	4	5	6	5	3	2	3

On Tuesday the tide follows the same pattern 1 hour later, and a storm keeps the water 0.5 meter deeper all day. Let $T(t)$ be the depth t hours after midnight on Tuesday.

(a) Write $T(t)$ in terms of D.

(b) A boat needs water at least 5.5 meters deep to tie up. At which of the times $t = 1, 3, 5, \ldots, 13$ on Tuesday can it tie up?

(c) Explain why $D(t+1) + 0.5$ would be the wrong rule.

ANSWER

Free sample from math.wiki for classroom use. Do not post. math.wiki

(a) $T(t) = D(t-1) + 0.5$ (b) $t = 3$, 5 and 7 (3 a.m., 5 a.m. and 7 a.m.) (c) $D(t+1) + 0.5$ moves the graph 1 to the left, so every depth would come 1 hour earlier, not later.

SOLUTION

(a) At time t on Tuesday the tide is where Monday's was 1 hour before, at $t - 1$, and the water is 0.5 m deeper: $T(t) = D(t-1) + 0.5$. Its graph is Monday's moved 1 right and 0.5 up.

(b) $T(t) \geq 5.5$ means $D(t-1) + 0.5 \geq 5.5$, so $D(t-1) \geq 5$. In the table D is at least 5 at 2, 4 and 6, so $t - 1 = 2, 4, 6$ and $t = 3, 5, 7$. Indeed $T(3) = 5.5$, $T(5) = 6.5$ and $T(7) = 5.5$, while $T(1) = 4.5$, $T(9) = 3.5$, $T(11) = 2.5$ and $T(13) = 3.5$ are too shallow.

(c) $D(t+1)$ at time t gives the depth Monday had 1 hour later. That is $f(x - h)$ with $h = -1$, a shift 1 to the left: each high and low tide would arrive 1 hour earlier than on Monday instead of 1 hour later.

Free sample from math.wiki for classroom use. Do not post. math.wiki