Adding and multiplying complex numbers

The square root of negative one is now a number. Can you still add and multiply the way you always have?

WORDS TO KNOW

Complex number: a number $a + bi$, with a and b real and $i^2 = -1$.

Real part and imaginary part: in $a + bi$, the real part is a and the imaginary part is b.

Standard form: the shape $a + bi$, real part first. Every answer goes back into this shape.

Like parts: real parts go with real parts, and imaginary parts go with imaginary parts, just as x terms go with x terms.

KEY IDEAS

1. Add and subtract part by part

Treat $a + bi$ like a binomial with i as the variable. Real parts combine with real parts, and imaginary parts with imaginary parts. To subtract, the minus sign changes the sign of **both** parts of the second number.

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$
$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

2. Multiply like two binomials, then replace i^2

Distribute each part of the first number over each part of the second. That gives four products, the same four as in $(a + bx)(c + dx)$. The one new step: i^2 is −1, so the i^2 term is real and joins the real part.

1. Distribute: four products.
2. Replace i^2 with −1.
3. Collect the real terms and the i terms, and write $a + bi$.

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

You do not need to learn the last line by heart. Expanding and replacing i^2 gets there every time. For a square, use the square of a binomial:

$$(a + bi)^2 = a^2 + 2abi + b^2i^2 = (a^2 - b^2) + 2abi$$

3. Check a complex solution by substituting

To test whether a complex number solves an equation, put it in for x and do the arithmetic. When one side is 0, the other side must come out with real part 0 **and** imaginary part 0.

WATCH OUT

- **Do not leave i^2 in the answer.** A result with i^2 in it is not in standard form yet. Replace it with −1.
- **Keep the middle term when you square.** $(5 + i)^2$ is not $25 + i^2$. It is $25 + 10i + i^2 = 24 + 10i$.
- **The parts stay apart in a sum.** $4 + 3i$ cannot be shortened, just as $4 + 3x$ cannot.
- **Write roots of negatives with i first.** Change $\sqrt{-9}$ to $3i$ before you multiply, because $\sqrt{a}\sqrt{b} = \sqrt{ab}$ fails when both numbers are negative.

Free sample from math.wiki for classroom use. Do not post. math.wiki

WORKED EXAMPLES

Example 1. Simplify $(7 + 2i) - (3 - 5i)$.

1. The minus sign changes the sign of both parts of the second number: $-(3 - 5i) = -3 + 5i$.
2. So the expression is $7 + 2i - 3 + 5i$.
3. Collect the real parts and the imaginary parts: $(7 - 3) + (2 + 5)i = 4 + 7i$.

Answer: $4 + 7i$. **Check:** add back: $(4 + 7i) + (3 - 5i) = 7 + 2i$.

Example 2. Multiply $(2 - i)(3 + 7i)$.

1. Distribute: four products.
$$(2 - i)(3 + 7i) = 6 + 14i - 3i - 7i^2$$
2. Replace i^2 with –1: $-7i^2 = -7(-1) = 7$.
3. Collect: the real terms give 6 + 7 = 13, and the i terms give $14i - 3i = 11i$.

Answer: $13 + 11i$. **Check:** with the general line, $ac - bd = 2 \cdot 3 - (-1)(7) = 13$ and $ad + bc = 2 \cdot 7 + (-1)(3) = 11$.

Example 3. Show that $x = 2 + 8i$ is a solution of $x^2 - 4x + 68 = 0$.

1. Work out each term on its own. The square:
$$(2 + 8i)^2 = 4 + 32i + 64i^2 = 4 + 32i - 64 = -60 + 32i$$
2. The middle term: $-4(2 + 8i) = -8 - 32i$.
3. Add the three pieces, real parts and imaginary parts apart.
$$(-60 + 32i) + (-8 - 32i) + 68 = (-60 - 8 + 68) + (32 - 32)i = 0$$

Answer: both parts are 0, so $x = 2 + 8i$ is a solution.

Free sample from math.wiki for classroom use. Do not post. math.wiki