Algebra 2 Unit 2 worksheet: Adding, subtracting and multiplying complex numbers (answer key)

ANSWERS AT A GLANCE

1. (a) $3+6i$ (b) $-9+5i$ (c) $-1+i$ (d) $3+4i$
2. (a) $-12-8i$ (b) 24 (c) $7+17i$ (d) $-2+23i$ (e) $6+6i$
3. (a) $15+8i$ (b) $5-12i$ (c) $2+2i\sqrt{3}$ (d) -4
4. (a) She squared each part and left out the middle term $2(4)(-3i)=-24i$; the square is $7-24i$. (b) He replaced $-5i^2$ with -5 instead of $+5$; the product is $7+3i$.
5. $z=-2+2i$
6. $f(-3-2i)=28+20i$
7. (a) $5+i$ (b) $-5+14i$ (c) $-5+14i$ (d) $(z+w)(z-w)=z^2-w^2$ uses only the distributive and commutative properties, which hold for complex numbers.
8. (a) $k=4$, square $32i$; $k=-4$, square $-32i$ (b) $k=2$ or $k=6$

WORKED SOLUTIONS

1. Add or subtract. Write each answer in standard form $a+bi$.

(a) $(8-3i)+(-5+9i)$ (b) $(-2+4i)-(7-i)$ (c) $\left(\frac{1}{2}+\frac{3}{4}i\right)-\left(\frac{3}{2}-\frac{1}{4}i\right)$

(d) $2(3-i)-3(1-2i)$

ANSWER

(a) $3+6i$ (b) $-9+5i$ (c) $-1+i$ (d) $3+4i$

SOLUTION

(a) Real parts 8 + (−5) = 3, imaginary parts −3 + 9 = 6: $3+6i$.

(b) The minus sign changes both parts of the second number: $-2+4i-7+i=(-2-7)+(4+1)i=-9+5i$.

(c) Real parts $\frac{1}{2}-\frac{3}{2}=-1$, imaginary parts $\frac{3}{4}+\frac{1}{4}=1$: $-1+i$.

(d) Distribute the real numbers first: $6-2i-3+6i=3+4i$.

2. Multiply. Write each answer in standard form $a+bi$.

(a) $4i(-2+3i)$ (b) $(6i)(-4i)$ (c) $(5+i)(2+3i)$ (d) $(-3+2i)(4-5i)$

(e) $\sqrt{-4}\left(3-\sqrt{-9}\right)$

ANSWER

(a) $-12-8i$ (b) 24 (c) $7+17i$ (d) $-2+23i$ (e) $6+6i$

SOLUTION

(a) $-8i+12i^2=-8i-12=-12-8i$, real part first.

(b) $-24i^2=-24(-1)=24$, a real number.

(c) $10+15i+2i+3i^2=10+17i-3=7+17i$.

(d) $-12+15i+8i-10i^2=-12+23i+10=-2+23i$.

(e) Write the roots with i first: $\sqrt{-4}=2i$ and $\sqrt{-9}=3i$. Then $2i(3-3i)=6i-6i^2=6+6i$.

3. Write each square or power in standard form.

(a) $(4+i)^2$ (b) $(2i-3)^2$ (c) $\left(\sqrt{3}+i\right)^2$ (d) $(1-i)^4$

ANSWER

(a) $15+8i$ (b) $5-12i$ (c) $2+2i\sqrt{3}$ (d) -4

SOLUTION

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(a) $16 + 8i + i^2 = 16 + 8i - 1 = 15 + 8i$.

(b) In standard form the number is $-3 + 2i$: $9 - 12i + 4i^2 = 9 - 12i - 4 = 5 - 12i$.

(c) $3 + 2i\sqrt{3} + i^2 = 3 + 2i\sqrt{3} - 1 = 2 + 2i\sqrt{3}$.

(d) Square twice. $(1 - i)^2 = 1 - 2i + i^2 = -2i$, so $(1 - i)^4 = (-2i)^2 = 4i^2 = -4$.

4. Each student made one mistake. Say what it is, and give the correct answer in standard form.

(a) Isabel wrote $(4 - 3i)^2 = 4^2 + (3i)^2 = 16 - 9 = 7$.

(b) Arjun wrote $(2 + 5i)(1 - i) = 2 - 2i + 5i - 5i^2 = 2 + 3i - 5 = -3 + 3i$.

ANSWER

(a) She squared each part and left out the middle term $2(4)(-3i) = -24i$; the square is $7 - 24i$. (b) He replaced $-5i^2$ with −5 instead of + 5; the product is $7 + 3i$.

SOLUTION

(a) A square of a sum is not the sum of the squares: $(p - q)^2 = p^2 - 2pq + q^2$ has a middle term. With $p = 4$ and $q = 3i$:

$$(4 - 3i)^2 = 16 - 24i + 9i^2 = 16 - 24i - 9 = 7 - 24i$$

Isabel's real part, 16 − 9 = 7, was right; the missing $-24i$ is the whole imaginary part.

(b) The four products are right. The slip is in the last step: $-5i^2 = -5(-1) = +5$, so

$$2 + 3i + 5 = 7 + 3i$$

5. Find the complex number z for which $2z - (3 - i) = i(5 + 7i)$. Check your answer in the original equation.

ANSWER

$z = -2 + 2i$

SOLUTION

First put the right side in standard form: $i(5 + 7i) = 5i + 7i^2 = -7 + 5i$.

Add $3 - i$ to both sides, adding real parts and imaginary parts:

$$2z = (-7 + 3) + (5 - 1)i = -4 + 4i$$

Divide both parts by 2: $z = -2 + 2i$.

Check: $2(-2 + 2i) - (3 - i) = -4 + 4i - 3 + i = -7 + 5i$, which is $i(5 + 7i)$.

6. Let $f(x) = x^2 - 4x + 11$. Find $f(-3 - 2i)$ and write it in standard form.

ANSWER

$f(-3 - 2i) = 28 + 20i$

SOLUTION

Substitute $x = -3 - 2i$ and work out each term on its own. The square, using $i^2 = -1$:

$$(-3 - 2i)^2 = 9 + 12i - 4 = 5 + 12i$$

The middle term: $-4(-3 - 2i) = 12 + 8i$.

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Add the two and the constant 11, real parts together and imaginary parts together:

$$f(-3-2i) = (5+12+11) + (12+8)i = 28+20i$$

7. Let $z = 2+3i$ and $w = 1-i$. Write each in standard form.

(a) zw (b) $z^2 - w^2$ (c) $(z+w)(z-w)$

(d) Explain why (b) and (c) must come out the same.

ANSWER

(a) $5+i$ (b) $-5+14i$ (c) $-5+14i$ (d) $(z+w)(z-w) = z^2 - w^2$ uses only the distributive and commutative properties, which hold for complex numbers.

SOLUTION

(a) $zw = (2+3i)(1-i) = 2-2i+3i-3i^2 = 2+i+3 = 5+i$.

(b) $z^2 = 4+12i+9i^2 = -5+12i$ and $w^2 = 1-2i+i^2 = -2i$, so $z^2 - w^2 = -5+12i+2i = -5+14i$.

(c) $z+w = 3+2i$ and $z-w = 1+4i$, so $(3+2i)(1+4i) = 3+12i+2i+8i^2 = 3+14i-8 = -5+14i$.

(d) Expanding $(z+w)(z-w)$ gives $z^2 - zw + wz - w^2$, and $-zw+wz = 0$ because multiplying complex numbers is commutative. So $(z+w)(z-w) = z^2 - w^2$ for any complex z and w, just as for real numbers.

8. (a) Find every real number k for which $(k+4i)^2$ is pure imaginary, and give the square for each one.

(b) Find every real number k for which the real part of $(k+4i)^2$ minus its imaginary part equals -28.

ANSWER

(a) $k = 4$, square $32i$; $k = -4$, square $-32i$ (b) $k = 2$ or $k = 6$

SOLUTION

Expand with k left as a letter:

$$(k+4i)^2 = k^2 + 8ki + 16i^2 = (k^2 - 16) + 8k\,i$$

Since k is real, $k^2 - 16$ is the real part and $8k$ is the imaginary part.

(a) Pure imaginary means real part 0 and imaginary part not 0. $k^2 - 16 = 0$ gives $k = 4$ or $k = -4$, and neither makes $8k$ zero. For $k = 4$ the square is $8(4)i = 32i$; for $k = -4$ it is $-32i$.

(b) The real part minus the imaginary part is $k^2 - 16 - 8k$. Set it equal to -28 and move 16 across:

$$k^2 - 8k = -12$$

Complete the square by adding 16 to both sides:

$$(k-4)^2 = 4, \qquad k - 4 = \pm 2$$

So $k = 2$ or $k = 6$, both real.

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