Synthetic division

Dividing by $x - c$ by long division writes the same things again and again. How do you do it with just one row of numbers?

WORDS TO KNOW

Synthetic division: a short way to divide a polynomial by $x - c$, using only the coefficients.

Coefficients: the numbers in front of each power, written highest power first.

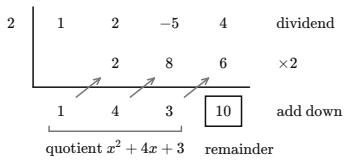
Bottom row: the quotient's coefficients, then the remainder in the last place.

Corner number: the number c from the divisor $x - c$, written at the top left.

Placeholder zero: a 0 written for a missing power, so every power keeps its column.

KEY IDEAS

1. Bring down, multiply, add



Synthetic division is long division with the letters taken out. Each "take away" in long division turns into an "add", which is why the corner holds c and not $-c$.

1. Write c in the corner and the dividend's coefficients along the top, with 0 for any missing power.
2. Bring the first coefficient straight down.
3. Multiply the newest bottom number by c, write the product under the next coefficient, and add down.
4. Repeat to the end. The last number is the **remainder**. The numbers before it are the **quotient**'s coefficients, starting **one power lower** than the dividend.

2. Getting the corner number right

- For $x - 3$, the corner is 3.
- For $x + 2$, write $x - (-2)$: the corner is −2.
- For $ax - b$, like $2x - 1$, use the corner $\frac{b}{a}$, then **divide the quotient by** a. The remainder stays the same.

3. What the answer says

$$p(x) = (x - c)\,q(x) + r$$

The remainder r is always a single number. A remainder of 0 means $x - c$ is a factor.

WATCH OUT

- **The divisor must be linear.** For a divisor like $x^2 + 1$, use long division. (If a quadratic divisor factors, you can divide by one linear factor at a time.)
- **Do not forget placeholder zeros.** $x^4 - 2x^3 + 5x - 8$ has no x^2 term, so its row is 1, −2, 0, 5, −8.
- **Watch the sign of the corner.** Dividing by $x + 4$ uses −4, not 4.
- **For $ax - b$, divide only the quotient by a.** The remainder is not divided.

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WORKED EXAMPLES

Example 1. Divide $x^4 - 2x^3 + 5x - 8$ by $x - 3$.

1. The divisor is $x - 3$, so the corner is 3. There is no x^2 term, so the top row is 1, −2, 0, 5, −8.

3	1	−2	0	5	−8
		3	3	9	42
	1	1	3	14	34

2. Bring down 1. Then $1 \cdot 3 = 3$ and $-2 + 3 = 1$. Next $1 \cdot 3 = 3$ and $0 + 3 = 3$. Next $3 \cdot 3 = 9$ and $5 + 9 = 14$. Last $14 \cdot 3 = 42$ and $-8 + 42 = 34$.
3. The dividend has degree 4, so the quotient has degree 3.

Answer: quotient $x^3 + x^2 + 3x + 14$, remainder 34.

Check: $(x - 3)(x^3 + x^2 + 3x + 14) = x^4 - 2x^3 + 5x - 42$, and adding 34 gives the dividend.

Example 2. Divide $x^3 + 5x^2 - 2x + 3$ by $x + 6$.

1. Write $x + 6$ as $x - (-6)$, so the corner is −6. The top row is 1, 5, −2, 3.

−6	1	5	−2	3
		−6	6	−24
	1	−1	4	−21

2. Bring down 1. Then $1(-6) = -6$ and $5 + (-6) = -1$. Next $(-1)(-6) = 6$ and $-2 + 6 = 4$. Last $4(-6) = -24$ and $3 + (-24) = -21$.

Answer: quotient $x^2 - x + 4$, remainder −21.

Check: $(x + 6)(x^2 - x + 4) = x^3 + 5x^2 - 2x + 24$, and $24 + (-21) = 3$.

Example 3. Divide $2x^3 + x^2 - 13x + 20$ by $2x + 7$.

1. Write $2x + 7 = 2\left(x + \frac{7}{2}\right)$, so divide by $x - \left(-\frac{7}{2}\right)$ first. The corner is $-\frac{7}{2}$.

$-\frac{7}{2}$	2	1	−13	20
		−7	21	−28
	2	−6	8	−8

2. Bring down 2. Then $2 \cdot \left(-\frac{7}{2}\right) = -7$ and $1 + (-7) = -6$. Next $(-6)\left(-\frac{7}{2}\right) = 21$ and $-13 + 21 = 8$. Last $8\left(-\frac{7}{2}\right) = -28$ and $20 + (-28) = -8$.
3. So dividing by $x + \frac{7}{2}$ gives $2x^2 - 6x + 8$, remainder −8. The divisor $2x + 7$ is twice as big, so halve the quotient: $x^2 - 3x + 4$. The remainder stays −8.

Answer: quotient $x^2 - 3x + 4$, remainder −8.

Check: $(2x + 7)(x^2 - 3x + 4) = 2x^3 + x^2 - 13x + 28$, and $28 + (-8) = 20$.

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