Algebra 2 Unit 3 worksheet: Synthetic division (answer key)

ANSWERS AT A GLANCE

1. (a) $c = -4$; 1, 0, −5, 2 (b) $c = -1$; 1, 0, 0, −3, 0, 0 (c) $c = -\frac{1}{2}$; 6, 0, 5, −1 (d) $c = -2$; −1, 0, 0, 7, 0
2. Quotient $x^2 + 4x + 4$, remainder 4
3. Quotient $-2x^3 - 4x^2 - 6x - 12$, remainder −36
4. Quotient $2x^2 + 5x + 5$, remainder −1
5. (a) She put 2 in the corner. Since $x + 2 = x - (-2)$, the corner number is −2; her tableau divides by $x - 2$. (b) Quotient $3x^2 - 2x - 1$, remainder 3.
6. (a) $q_1(x) = x^2 - 2x + 3$, $r_1 = 5$ (b) $q_2(x) = x$, $r_2 = 3$ (c) quotient 1, $r_3 = 2$ (d) $p(x) = (x-2)^3 + 2(x-2)^2 + 3(x-2) + 5$, so $A = 2$, $B = 3$, $C = 5$.
7. Corner −2; top row 3, 11, 6, −2, 1; middle row −6, −10, 8, −12; bottom row 3, 5, −4, 6, −11. Divisor $x + 2$, dividend $3x^4 + 11x^3 + 6x^2 - 2x + 1$, quotient $3x^3 + 5x^2 - 4x + 6$, remainder −11.

WORKED SOLUTIONS

1. Set up each synthetic division, but do not divide: write the corner number c and the top row of coefficients, highest power first.

(a) $(x^3 - 5x + 2) \div (x + 4)$ (b) $(x^5 - 3x^2) \div (x + 1)$ (c) $(6x^3 + 5x - 1) \div (2x + 1)$ (d) $(7x - x^4) \div (x + 2)$

ANSWER

(a) $c = -4$; 1, 0, −5, 2 (b) $c = -1$; 1, 0, 0, −3, 0, 0 (c) $c = -\frac{1}{2}$; 6, 0, 5, −1 (d) $c = -2$; −1, 0, 0, 7, 0

SOLUTION

(a) $x + 4 = x - (-4)$, so $c = -4$. The x^2 term is missing: 1, 0, −5, 2.

(b) $x + 1 = x - (-1)$, so $c = -1$. Degree 5 needs six columns; only x^5 and x^2 appear: 1, 0, 0, −3, 0, 0. The last 0 is the constant term.

(c) $2x + 1 = 2\left(x + \frac{1}{2}\right) = 2\left(x - \left(-\frac{1}{2}\right)\right)$, so $c = -\frac{1}{2}$. The x^2 term is missing: 6, 0, 5, −1. (After dividing, the quotient is divided by 2.)

(d) $c = -2$. Reorder: $-x^4 + 0x^3 + 0x^2 + 7x + 0$, so −1, 0, 0, 7, 0.

2. Use synthetic division to divide $x^3 + 9x^2 + 24x + 24$ by $x + 5$. Give the quotient and the remainder.

ANSWER

Quotient $x^2 + 4x + 4$, remainder 4

SOLUTION

Since $x + 5 = x - (-5)$, the corner number is $c = -5$. No power is missing.

-5	1	9	24	24
		-5	-20	-20
	1	4	4	4

Bring down 1, then multiply each new bottom number by −5 and add down. The dividend has degree 3, so the quotient has degree 2: it is $x^2 + 4x + 4$, and the remainder is 4.

Check: $(x + 5)(x^2 + 4x + 4) + 4$ multiplies out to the dividend.

3. Use synthetic division to divide $-2x^4 + 2x^2 - 12$ by $x - 2$. Give the quotient and the remainder.

ANSWER

Quotient $-2x^3 - 4x^2 - 6x - 12$, remainder −36

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SOLUTION

Here $c = 2$. The x^3 and x terms are missing, so each gets a 0: the top row is −2, 0, 2, 0, −12.

$$\begin{array}{c|ccccc} 2 & -2 & 0 & 2 & 0 & -12 \\ & & -4 & -8 & -12 & -24 \\ \hline & -2 & -4 & -6 & -12 & -36 \end{array}$$

The dividend has degree 4, so the quotient has degree 3: $-2x^3 - 4x^2 - 6x - 12$, with remainder −36.

Without the two zeros the tableau would have only three columns before the remainder, and every number after the first would land under the wrong power.

4. Use synthetic division to divide $6x^3 + 13x^2 + 10x - 6$ by $3x - 1$. Give the quotient and the remainder.

ANSWER

Quotient $2x^2 + 5x + 5$, remainder −1

SOLUTION

Write $3x - 1 = 3\left(x - \frac{1}{3}\right)$ and divide by $x - \frac{1}{3}$ first, with $c = \frac{1}{3}$:

$$\begin{array}{c|cccc} \frac{1}{3} & 6 & 13 & 10 & -6 \\ & & 2 & 5 & 5 \\ \hline & 6 & 15 & 15 & -1 \end{array}$$

The quotient by $x - \frac{1}{3}$ is $6x^2 + 15x + 15$, remainder −1. Divide the quotient, not the remainder, by 3: the quotient by $3x - 1$ is $2x^2 + 5x + 5$ and the remainder is −1.

Check: $(3x - 1)(2x^2 + 5x + 5) - 1$ multiplies out to the dividend.

5. Layla divided $3x^3 + 4x^2 - 5x + 1$ by $x + 2$. Her tableau is below, and she wrote that the quotient is $3x^2 + 10x + 15$ with remainder 31.

$$\begin{array}{c|cccc} 2 & 3 & 4 & -5 & 1 \\ & & 6 & 20 & 30 \\ \hline & 3 & 10 & 15 & 31 \end{array}$$

(a) What did Layla do wrong? Her arithmetic is correct, so her tableau does divide $3x^3 + 4x^2 - 5x + 1$ by some divisor: which one?

(b) Divide by $x + 2$ correctly, and check your answer by multiplying.

ANSWER

(a) She put 2 in the corner. Since $x + 2 = x - (-2)$, the corner number is −2; her tableau divides by $x - 2$. (b) Quotient $3x^2 - 2x - 1$, remainder 3.

SOLUTION

(a) The tableau divides by $x - c$, where c is the corner number. Layla used $c = 2$, so she divided by $x - 2$, and for $x - 2$ her quotient $3x^2 + 10x + 15$ and remainder 31 are right. For $x + 2 = x - (-2)$ the corner must be −2.

(b) With $c = -2$:

$$\begin{array}{c|cccc} -2 & 3 & 4 & -5 & 1 \\ & & -6 & 4 & 2 \\ \hline & 3 & -2 & -1 & 3 \end{array}$$

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The quotient is $3x^2 - 2x - 1$ and the remainder is 3.

Check: $(x+2)(3x^2-2x-1) = 3x^3 - 2x^2 - x + 6x^2 - 4x - 2 = 3x^3 + 4x^2 - 5x - 2$, and −2 + 3 = 1, so adding the remainder gives the dividend.

6. Let $p(x) = x^3 - 4x^2 + 7x - 1$.

(a) Divide $p(x)$ by $x - 2$. Call the quotient $q_1(x)$ and the remainder r_1.

(b) Divide $q_1(x)$ by $x - 2$. Call the quotient $q_2(x)$ and the remainder r_2.

(c) Divide $q_2(x)$ by $x - 2$. Call the remainder r_3.

(d) Use your three divisions to write $p(x)$ in powers of $(x - 2)$, as $p(x) = (x-2)^3 + A(x-2)^2 + B(x-2) + C$. Explain how the numbers A, B and C come from your work.

ANSWER

(a) $q_1(x) = x^2 - 2x + 3$, $r_1 = 5$ (b) $q_2(x) = x$, $r_2 = 3$ (c) quotient 1, $r_3 = 2$ (d) $p(x) = (x-2)^3 + 2(x-2)^2 + 3(x-2) + 5$, so $A = 2$, $B = 3$, $C = 5$.

SOLUTION

(a) With $c = 2$:

2	1	−4	7	−1
		2	−4	6
	1	−2	3	5

So $q_1(x) = x^2 - 2x + 3$ and $r_1 = 5$.

(b)

2	1	−2	3
		2	0
	1	0	3

The bottom 0 is a real coefficient: $q_2(x) = 1x + 0 = x$, and $r_2 = 3$.

(c)

2	1	0
		2
	1	2

The quotient is 1 and $r_3 = 2$.

(d) Each division says dividend $= (x-2) \cdot$ quotient $+$ remainder:

$$p(x) = (x-2)q_1(x) + 5, \quad q_1(x) = (x-2)q_2(x) + 3, \quad q_2(x) = (x-2)\cdot 1 + 2$$

Substitute from the last line up: $q_1(x) = (x-2)\big((x-2)+2\big) + 3 = (x-2)^2 + 2(x-2) + 3$, so

$$p(x) = (x-2)^3 + 2(x-2)^2 + 3(x-2) + 5$$

The remainders, last to first, are $A = r_3 = 2$, $B = r_2 = 3$ and $C = r_1 = 5$.

Check: $(x-2)^3 = x^3 - 6x^2 + 12x - 8$ and $2(x-2)^2 = 2x^2 - 8x + 8$; adding $3x - 6$ and 5 gives $x^3 - 4x^2 + 7x - 1$.

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7. Some entries of this synthetic division were smudged; each $\square$ is one lost number. Fill in every entry, then write the divisor, the dividend, the quotient and the remainder.

$\square$	3	11	$\square$	-2	$\square$
		-6	$\square$	8	$\square$
	3	$\square$	-4	$\square$	-11

ANSWER

Corner -2; top row 3, 11, 6, −2, 1; middle row −6, −10, 8, −12; bottom row 3, 5, −4, 6, −11. Divisor $x + 2$, dividend $3x^4 + 11x^3 + 6x^2 - 2x + 1$, quotient $3x^3 + 5x^2 - 4x + 6$, remainder −11.

SOLUTION

Each middle number is the corner number times the bottom number one column to its left. The first is $-6 = c \cdot 3$, so $c = -2$.

Second column: 11 + (−6) = 5. Its product is (−2)(5) = −10.

Third column: the top number plus −10 is −4, so it is 6. Its product is (−2)(−4) = 8, which matches the 8 shown.

Fourth column: −2 + 8 = 6. Its product is (−2)(6) = −12.

Last column: the top number plus −12 is −11, so it is 1.

-2	3	11	6	-2	1
		-6	-10	8	-12
	3	5	-4	6	-11

The corner -2 means the divisor is $x - (-2) = x + 2$. The top row has five numbers, so the dividend is the quartic $3x^4 + 11x^3 + 6x^2 - 2x + 1$; the quotient is $3x^3 + 5x^2 - 4x + 6$ and the remainder is -11.

Check: $(x + 2)(3x^3 + 5x^2 - 4x + 6) = 3x^4 + 11x^3 + 6x^2 - 2x + 12$, and 12 + (−11) = 1.

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