Calculus 1 Unit 1 quiz 4: Infinite limits and vertical asymptotes, Discontinuities and how to remove them, Making a piecewise function continuous

Name _____ Date _____ Period _____

8 questions, 19 points

1. Let $g(x) = \dfrac{x^2 - 6}{x^2 + 2x - 15}$. Find $\lim_{x \to 3^-} g(x)$ and $\lim_{x \to 3^+} g(x)$, and explain each sign. (2 points)

2. Let $k(x) = \dfrac{e^x + 1}{|x|\,(x - 3)}$. (2 points)

(a) Find $\lim_{x \to 0} k(x)$, or explain why it does not exist.

(b) Find $\lim_{x \to 3^-} k(x)$ and $\lim_{x \to 3^+} k(x)$.

3. Let $f(x) = \dfrac{x^2 - 16}{2|x| - 8}$. Find and classify every discontinuity of f. Then write the continuous extension of f as one formula that holds for every real x. (2 points)

4. Find the constant k that makes f continuous, and the height at which the pieces meet.

$$f(x) = \begin{cases} kx + 19, & x < 3 \\ x^2 + k, & x \geq 3 \end{cases}$$

(2 points)

5. Samir writes: "$\lim\limits_{x \to 2} \dfrac{x^2 - 4}{(x - 2)^2} = \infty$, because the bottom $(x - 2)^2$ tends to 0 and is positive on both sides of 2."

Explain what is wrong with his reasoning, and find the correct one-sided limits at 2 and the two-sided limit. (3 points)

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6. Let $f(x) = 2x - 1$ for $x < 3$ and $f(x) = x^2 - 7$ for $x \geq 3$. (3 points)

(a) Find $\lim_{x \to 3^-} f(x)$ and $\lim_{x \to 3^+} f(x)$.

(b) Find every constant c for which $(x^2 - c)f(x)$ is continuous at 3.

(c) Find the size of the jump of $(x + 2)f(x)$ at 3.

7. Find a and b so that f is continuous on its whole domain $[-3, \infty)$.

$$f(x) = \begin{cases} a\sqrt{x+3}, & -3 \leq x < 1 \\ bx^2 + a, & 1 \leq x \leq 2 \\ 21 - 3x, & x > 2 \end{cases}$$

(3 points)

8. Let $f(x) = \dfrac{x^2 - 9}{x^3 - 6x^2 + 9x}$. Find and classify every discontinuity of f, giving the one-sided limits at each. Explain why the discontinuity at 3 is not removable, even though a factor $x - 3$ cancels. (2 points)

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