Calculus 1 Unit 1 quiz 5: Limits at infinity and horizontal asymptotes, Comparing growth rates at infinity

Name _____ Date _____ Period _____

8 questions, 22 points

1. Find the limit below.

$$\lim_{x \to -\infty} \frac{(3x-5)(6x+2)}{(2x+8)^2}$$

(2 points)

2. Find the constant c for which

$$\lim_{x \to \infty} \frac{c \cdot 4^x + x^6}{2^{2x+4} - x \ln x} = 3.$$

(2 points)

3. Find the horizontal asymptote at each end of the graph of $f(x) = \dfrac{\sqrt{x^2+5}}{\sqrt{4x^2-x}+x}$. (3 points)

4. Let $f(x) = \dfrac{5x^2}{x^2+4}$, so that $\lim_{x \to \infty} f(x) = 5$. (3 points)

(a) Show that $|f(x) - 5| = \dfrac{20}{x^2+4}$ for every x.

(b) In the definition of this limit, find the smallest N that works for the tolerance $t = 0.05$. Give it exactly.

(c) For $0 < t < 5$, find an N in terms of t that works.

5. Find the constant c for which

$$\lim_{x \to \infty} \left(\sqrt{x^2 + cx} - \sqrt{x^2 - 4x + 1} \right) = 3.$$

(3 points)

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6. Let $f(x) = x^3 e^{1/x}$ for $x \neq 0$. Find $\lim_{x \to 0^+} f(x)$ and $\lim_{x \to 0^-} f(x)$, and decide whether $\lim_{x \to 0} f(x)$ exists. (3 points)

7. Functions f, g and h are positive for all large x, and all that is known about them is

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0 \quad \text{and} \quad \lim_{x \to \infty} \frac{h(x)}{g(x)} = 4.$$

(a) Find $\lim_{x \to \infty} \dfrac{f(x) + 3h(x)}{g(x) + h(x)}$.

(b) Does f grow faster than h, slower, or at the same rate? Justify with a limit. (3 points)

8. Rank these four functions from slowest growing to fastest. Rewrite each logarithm first, then justify each neighboring pair in your ranking with the limit of a quotient.

$$\ln(x^x), \quad x^{0.2}, \quad \ln(2^x), \quad \ln(x^{10})$$

(3 points)

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