Calculus 1 Unit 1 quiz 6: The intermediate value theorem, The epsilon delta definition of a limit

Name _____ Date _____ Period _____

7 questions, 20 points

1. A function f is continuous on $[1, 4]$, with $f(1) = 2$ and $f(4) = 8$. Show that $f(c) = c^2$ for some c strictly between 1 and 4. (2 points)

2. The limit $\lim_{x \to 3}(x^2 - 4x) = -3$ holds. For $\varepsilon = 1$, find the largest δ that works. Give it exactly. (2 points)

3. Show that the equation $\sqrt{x + 4} = x - 5$ has a solution strictly between 8 and 9. Give a complete justification with the intermediate value theorem. (3 points)

4. Elena's greenhouse controller measures the air temperature C in degrees Celsius and displays it in degrees Fahrenheit, $F(C) = \frac{9}{5}C + 32$. At 25 degrees Celsius it displays 77 degrees Fahrenheit. (3 points)

(a) Prove from the definition that $\lim_{C \to 25} F(C) = 77$.

(b) Elena wants the display within 0.9 degrees Fahrenheit of 77. Use your rule from (a) to find how close to 25 degrees Celsius the air temperature must be to guarantee this.

5. This is the scratch work for a proof that $\lim_{x \to -1}(2x^2 - x) = 3$. (3 points)

(a) Factor $2x^2 - x - 3$, and write $|2x^2 - x - 3|$ as $|x + 1|$ times another factor.

(b) Agree that $\delta \leq 1$. Find a number K with $|2x - 3| < K$ for every x with $|x + 1| < 1$.

(c) Write a δ in terms of ε that the proof can use, and give its value for $\varepsilon = 0.14$.

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6. Let $f(x) = \dfrac{x^2 + 5}{x - 1}$. Then $f(2) = 9$ and $f(5) = \dfrac{15}{2}$. (4 points)

(a) Use the intermediate value theorem to show that $f(c) = 8$ for some c in $[2, 5]$.

(b) Andre says: "7 is not between $f(2)$ and $f(5)$, so by the intermediate value theorem the equation $f(x) = 7$ has no solution in $[2, 5]$." Is his reasoning right? Solve $f(x) = 7$ on $[2, 5]$ to check.

7. Isabel uses bisection on $p(x) = x^3 + 2x^2 - 6$ over $[1, 2]$, where $p(1) = -3$ and $p(2) = 10$. Her first three rows are below, with values rounded to four decimal places. (3 points)

a	b	m	$p(m)$
1	2	1.5	1.875
1	1.5	1.25	−0.9219
1	1.25	1.125	−2.0449

(a) Find the first row that is wrong, and say what went wrong.

(b) Correct that row so that the bisection has three correct steps. Give the final interval and its midpoint.

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