Calculus 1 Unit 1 test: Limits and continuity

Name _____ Date _____ Period _____

12 questions, 49 points

1. Let $h(x) = \lfloor x \rfloor \, (x - 3)$. (4 points)

(a) Find $\lim_{x \to 3^-} h(x)$ and $\lim_{x \to 3^+} h(x)$. Does $\lim_{x \to 3} h(x)$ exist?

(b) Find $\lim_{x \to 5^-} h(x)$ and $\lim_{x \to 5^+} h(x)$. Does $\lim_{x \to 5} h(x)$ exist?

(c) Find every number a at which $\lim_{x \to a} h(x)$ does not exist, and explain why the limit exists everywhere else.

2. Find the constants a and b for which

$$\lim_{x \to -2} \frac{x^2 + ax + b}{x^2 - 4} = 3.$$

Explain why the numerator must be 0 at $x = -2$. (4 points)

3. Find $\lim_{x \to 0} \dfrac{\sin(x^2 + 4x)}{\tan(x^2 - 3x)}$. Name each limit result you use. (4 points)

4. Diego models the height of a delivery drone, in meters, t seconds after takeoff. The drone climbs, hovers, then comes down:

$$A(t) = \begin{cases} \frac{3}{5}t^2 & \text{if } 0 \le t < 10 \\ 60 & \text{if } 10 \le t \le 40 \\ \frac{(100 - t)^2}{50} & \text{if } 40 < t \le 100 \end{cases}$$

(4 points)

(a) Is A continuous at $t = 10$? Check each condition.

(b) Is A continuous at $t = 40$? Check each condition.

(c) A real drone's height cannot change by a jump. What does your answer to (b) say about Diego's model?

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5. Let $f(x) = \dfrac{x^2 - 9}{\sqrt{x+1} - 2}$ for $x \geq -1$ with $x \neq 3$, and $f(3) = 24$. Show that f is continuous on $[-1, \infty)$. (4 points)

6. For a constant c, let $f(x) = \dfrac{x^2 + cx - 6}{x^2 - 4}$. (4 points)

(a) Find the value of c for which the discontinuity at 2 is removable, and the value at 2 that removes it.

(b) Find the value of c for which the discontinuity at -2 is removable, and the value at -2 that removes it.

(c) Classify both discontinuities for every other value of c, and explain why no c makes both of them removable.

7. Find the constants a and b that make f continuous at every number, and explain why only one value of a can work.

$$f(x) = \begin{cases} \dfrac{x^2 + ax - 15}{x - 3}, & x < 3 \\ bx - 4, & x \geq 3 \end{cases}$$

(4 points)

8. Let $f(x) = \dfrac{x^2 + 1}{\sqrt{x^2 + 4}}$. Use limits to find the oblique asymptote of the graph of f at each end. (Long division does not apply, since f is not a rational function.) (4 points)

9. Find each limit.

(a) $\displaystyle\lim_{x \to \infty} \frac{\ln\left(x^6 + 4^x\right)}{\ln\left(x^6 + 2^x\right)}$ (b) $\displaystyle\lim_{x \to \infty} \frac{x^6 + 4^x}{x^6 + 2^x}$

(4 points)

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10. Let

$$f(x) = \begin{cases} x, & x < 1 \\ x + 2, & x \geq 1 \end{cases} \qquad g(x) = \begin{cases} 2x + 3, & x < 1 \\ x^2 + 2, & x \geq 1 \end{cases}$$

(a) Find $\lim_{x \to 1^-}$ and $\lim_{x \to 1^+}$ of $f(x)$ and of $g(x)$. Does $\lim_{x \to 1} f(x)$ exist? Does $\lim_{x \to 1} g(x)$? (4 points)

(b) Use the limit laws for one-sided limits to find $\lim_{x \to 1^-}$ and $\lim_{x \to 1^+}$ of $f(x) + g(x)$ and of $f(x)\, g(x)$. Which of $\lim_{x \to 1} \big[f(x) + g(x) \big]$ and $\lim_{x \to 1} f(x)\, g(x)$ exist?

(c) Explain why $\lim_{x \to 1} \big[f(x) + g(x) \big] = \lim_{x \to 1} f(x) + \lim_{x \to 1} g(x)$ cannot be written here.

11. Amara's fishing float is a wooden ball of radius 1 decimeter. In a made-up model for its wood, the float rests with its lowest point h decimeters below the water surface, where h solves the equation below.

$$h^3 - 3h^2 + 1 = 0$$

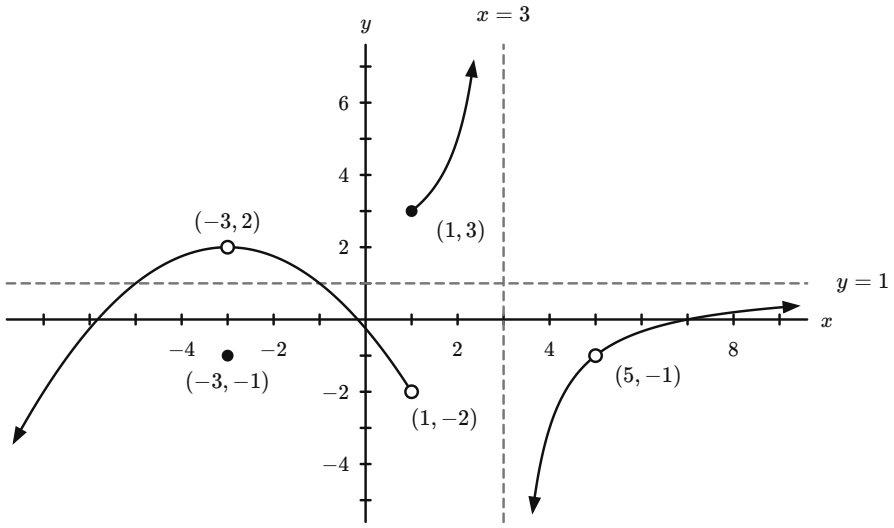
(a) Use the intermediate value theorem to show that the equation has a solution with $0 < h < 1$. (4 points)

(b) Take three bisection steps on $[0, 1]$. Use a calculator and round each value to four decimal places.

(c) Give your estimate of h (the last midpoint) and the most it can differ from the true depth.

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12. The graph of f is drawn below. The tick marks are 1 unit apart, the dashed lines are $x = 3$ and $y = 1$, and to the left of $x = 1$ the graph is part of a parabola. The arrows show the graph going on in the same way. (5 points)



(a) Find every a with $-7 < a < 9$ at which $\lim_{x \to a} f(x)$ does not exist. For each one, give or describe both one-sided limits.

(b) Find every a with $-7 < a < 9$ at which $\lim_{x \to a} f(x)$ exists but is not equal to $f(a)$, counting the case where $f(a)$ is undefined. Give the limit and $f(a)$ for each.

(c) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

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