Calculus 1 Unit 1 worksheet: The intermediate value theorem

Name _____ Date _____ Period _____

1. A function f is continuous on $[-2, 6]$. All that is known about it is this table.

x	-2	0	3	6
$f(x)$	3	-1	4	-5

(a) What is the least number of zeros f must have in $[-2, 6]$? Give an interval for each one.

(b) Must the equation $f(x) = -3$ have a solution in $[-2, 6]$? If so, in which interval between table entries?

(c) Must the equation $f(x) = 5$ have a solution in $[-2, 6]$? Explain.

2. Find the integer k for which the equation $x^3 + 4x = 89$ has a solution strictly between k and $k + 1$. Justify your answer with the intermediate value theorem, and explain why the equation has no other real solution.

3. Let $f(x) = 5 - x - x^4$.

(a) Show that f has a zero strictly between 1 and 2.

(b) Find the fewest bisection steps, starting from $[1, 2]$, that guarantee the midpoint of the last interval is within 0.05 of the zero.

(c) Carry out exactly that many steps. Use a calculator and round each value of f to four decimal places. For each step give the midpoint, the value of f there, and the interval you keep. Then give the final interval and your estimate of the zero.

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4. Kevin runs bisection on $f(x) = \tan x$ over $[1, 2]$, with x in radians. Use a calculator in radian mode and round each value to two decimal places.

(a) Show that $f(1)$ and $f(2)$ have opposite signs.

(b) Take three bisection steps. For each step give the midpoint, the value of f there, and the interval kept.

(c) Does $\tan x$ have a zero in $[1, 2]$? What number are Kevin's intervals closing in on? Explain why this does not contradict the intermediate value theorem.

5. Jasmine runs a loop route 10 km long that starts and ends at the same spot. Let $E(d)$ be her elevation, in meters, after she has run d km. Then E is continuous on $[0, 10]$ and $E(0) = E(10)$. Show that some two points on the loop that are 5 km apart along the route have the same elevation; that is, show that $E(c) = E(c + 5)$ for some c in $[0, 5]$.

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