Calculus 1 Unit 1 worksheet: Continuity at a point

Name _____ Date _____ Period _____

1. Use continuity to find each limit. Name the result that lets you substitute.

(a) $\lim_{x \to -1} (2x^3 - x + 4)$

(b) $\lim_{x \to 2} \dfrac{x^2 + 5}{3x - 1}$

(c) $\lim_{x \to 5} \sqrt{x^2 - 9}$

(d) $\lim_{x \to \pi} x \sin \dfrac{x}{2}$

(e) $\lim_{x \to 3} \sqrt[3]{1 - x^2}$

2. Let

$$f(x) = \begin{cases} 2x - 1 & \text{if } x < 2 \\ 5 & \text{if } x = 2 \\ x^2 - 1 & \text{if } x > 2 \end{cases}$$

Check each of the three conditions at 2 and decide whether f is continuous at 2.

3. Let $f(x) = \dfrac{\sqrt{x + 6} - 3}{x - 3}$ for $x \neq 3$, and $f(3) = \dfrac{1}{3}$. Find $\lim_{x \to 3} f(x)$, and decide whether f is continuous at 3. If it is not, name the condition that fails.

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4. Let $u(x) = 2$ for $x \geq 0$ and $u(x) = 3$ for $x < 0$. Let $v(x) = 3$ for $x \geq 0$ and $v(x) = 2$ for $x < 0$. Neither u nor v is continuous at 0.

(a) Which of $u + v$, $u - v$, uv and $\dfrac{u}{v}$ are continuous at 0? Check the conditions for each.

(b) Explain why the results for sums, products and quotients cannot settle part (a).

5. Let $r(x) = \dfrac{x + 1}{x^2 + 4x + c}$, where c is a constant.

(a) For which values of c is r continuous at every number?

(b) For which value of c is r discontinuous at exactly one number? Which number?

(c) Take $c = 3$. At which numbers is r discontinuous? Explain why canceling a common factor does not make r continuous at -1.

6. Decide whether each statement is true for all functions f and g and every number a. Prove a true one with the results of this lesson, and give a counterexample to a false one, checking the conditions.

(a) If g is continuous at a and f is continuous at a, then $f\big(g(x)\big)$ is continuous at a.

(b) If f is continuous at a and $f(a) > 0$, then $\sqrt{f(x)}$ is continuous at a.

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