Calculus 1 Unit 1 worksheet: Making a piecewise function continuous

Name _____ Date _____ Period _____

1. Find the constant k that makes each function continuous on its whole domain.

(a) $f(x) = \begin{cases} 3x + k, & x < 2 \\ x^2 - k, & x \geq 2 \end{cases}$

(b) $g(x) = \begin{cases} kx^2, & x \leq 3 \\ 2x + k, & x > 3 \end{cases}$

(c) $p(x) = \begin{cases} \dfrac{k}{x}, & x < -2 \\ x + 5, & x \geq -2 \end{cases}$

(d) $q(x) = \begin{cases} k\sqrt{x}, & 0 \leq x < 4 \\ x^2 - 2k, & x \geq 4 \end{cases}$

2. Layla designs a skateboard ramp. Its height is $h(x)$ centimeters at a horizontal distance of x meters from a mark on the ground:

$$h(x) = \begin{cases} 0, & 0 \leq x < 1 \\ a(x-1)^2, & 1 \leq x \leq 3 \\ bx - a, & 3 < x \leq 5 \end{cases}$$

The ramp starts on flat ground, curves up, then rises in a straight line to its lip at $x = 5$, which is 110 centimeters high. The surface may turn a corner, but it must never have a step.

(a) Explain why the boundary at $x = 1$ gives no equation for a. (b) Find a and b. (c) How high is the ramp where the curve meets the straight part?

3. Here the unknown constant is where the boundary sits. Let

$$f(x) = \begin{cases} x^2, & x < k \\ -2x + 8, & x \geq k \end{cases}$$

Find every boundary k that makes f continuous. For each one, give the height at which the two pieces meet.

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4. Let $f(x) = \dfrac{12}{x + k}$ for $x < 2$ and $f(x) = x^2$ for $x \geq 2$.

(a) Find the k that makes f continuous at 2. With that k, is f continuous at every number? If not, say where it fails and give the largest intervals on which f is continuous.

(b) Answer the same questions when the second rule is $x^2 - 7$ instead.

5. Find the constants c and k that make f continuous at 2, where

$$f(x) = \begin{cases} \dfrac{\frac{6}{x} - 3}{x - 2}, & 0 < x < 2 \\ c, & x = 2 \\ kx + 1, & x > 2 \end{cases}$$

6. Let $f(x) = 2\sin(kx)$ for $x < 3$ and $f(x) = x + c$ for $x \geq 3$, where $k > 0$ and c are constants (sine of an angle in radians).

(a) Take $c = -2$. Find every k with $0 < k < \pi$ that makes f continuous.

(b) Find every c for which at least one $k > 0$ makes f continuous.

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